A Bilattice-based GQT Framework for Interrogatives and Interrogative Inferences

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The studies on interrogatives in logic and formal semantics have been a difficult task because there is not an intuitive and uncontroversial notion of truth values for interrogatives. Thus we see different frameworks for interrogatives with different merits and demerits. In this paper, I will formulate a theoretical framework that combines Gutierrez-Rexach 's GQT-based framework (in [4, 5]) and Nelken and Francez's bilattice-based framework (in [7, 8]) for interrogatives and derive certain valid inferential patterns involving interrogatives based on this framework.

Gutierrez-Rexach's framework is based on Generalized Quantifier Theory (GQT) and treats a WH-word as an interrogative quantifier (IQ) that requires, in addition to the ordinary arguments, an "answer argument" to make a complete proposition. For instance, the truth condition of "who" is represented by "$who(Y)(X) = 1$ iff $PERSON \cap Y = X$", where X is the answer argument. Thus, under this approach the question "Who sang" is semantically equivalent to the noun phrase "person(s) who sang".

Nelken and Francez's framework assumes that interrogatives are of the same semantic type as that of propositions. The denotation of declaratives and interrogatives are thus both truth values. However, to distinguish the two types of sentences, they adopt 5 truth values which are arranged in 2 lattices (hence a "bilattice"). For declaratives, there are 3 truth values: t ("known to be true"), f ("known to be false") and uk ("unknown whether true or false"). For interrogatives, they borrow the concept of "resolvedness" from [1] and assume 2 truth values: r ("resolved") and ur ("unresolved"). The two groups of truth values are related by the resolvedness conditions of interrogatives. For illustration, consider the polar question "Did Mary kiss John" whose formal representation and resolvedness condition is "$\|?(KISS(m, j))\| = r$ iff $\|KISS(m, j)\| \in \{t, f\}$" (where $\|p\|$ denotes the truth value of p), meaning that "Did Mary kiss John" is resolved iff it is known whether Mary kissed John.

In this paper, I will develop a formal framework for interrogatives that is based on Nelken and Francez's framework but with substantial modification. The reason for choosing Nelken and Francez's framework as the basis is that their framework is extensional and is thus easier to manipulate than an intensional framework such as [2]. Moreover, since their framework has a clear definition for truth values for both declaratives and interrogatives, it is straightforward to define inferential relations between interrogatives and is thus convenient to study the issue of interrogative inferences under this framework.

Nevertheless, Gutierrez-Rexach's GQT-based framework also has its merits because WH-words do share certain characteristics with ordinary quantifiers. Under Gutierrez-Rexach's framework, certain phenomena related to interrogatives can be studied from the perspective of GQT. Moreover, it is found that IQs also possess certain properties that are thoroughly studied in GQT, such as conservativity, monotonicity, intersectivity, etc. For this reason, the framework proposed in this paper will also incorporate certain elements of Gutierrez-Rexach's framework.

Following the traditional GQT approach (such as in [6]), I will formulate the resolvedness conditions of IQs as set relations. But since there are now 3 truth values for declaratives, we first have to define new notions of sets as follows (in what follows, U represents the universe):

$$X_t = \{x \in U : \|x \in X\| = t\} \tag{1}$$

$$X_f = \{x \in U : \|x \in X\| = f\} \tag{2}$$

$$X_{uk} = \{x \in U : \|x \in X\| = uk\} \tag{3}$$

Thus, with respect to every concept X, we have 3 sets X_t, X_f and X_{uk} containing elements that are known to belong to X, known not to belong to X and unknown whether to belong to X, respectively.

We can now write down the resolvedness conditions of IQs using these notions. For example, the resolvedness conditions for "*who*", "(*everybody except who*)" and "*which*" are as follows:

$$\|who(-)(B)\| = r \Leftrightarrow (PERSON \cap B)_{uk} = \emptyset^1 \tag{4}$$

$$\|(\textit{everybody except who})(-)(B)\| = r \Leftrightarrow (PERSON - B)_{uk} = \emptyset \tag{5}$$

$$\|which(A)(B)\| = r \Leftrightarrow (A \cap B)_{uk} = \emptyset \tag{6}$$

Note that the above conditions treat the IQs "who", etc. as "strongly exhaustive IQs". Under this interpretation, the question "$who(-)(B)$" is resolved iff for every element x, it is known whether x is a person belonging to B. In other words, there is no element x such that it is not known whether x is a person belonging to B. This is represented by the set relation $(PERSON \cap B)_{uk} = \emptyset$. The resolvedness conditions of other IQs can also be formulated as $S_{uk} = \emptyset$ for an appropriate set S.

[1] In this paper, I adopt the notation in [6] that represents a quantified statement in the form of a tripartite structure $Q(A)(B)$ where Q, A and B represent the determiner, subject (excluding the determiner) and predicate of the sentence, respectively. When Q is a noun phrase (such as "who") instead of a determiner, the A argument is empty and is represented by "$-$". Thus, "Who sang" is represented as "$who(-)(SING)$".

We can also determine the constituent answer (CA) and sentential answer (SA) to a strongly exhaustive IQ as follows: let Q be a strongly exhaustive IQ whose resolvedness condition has the form $S_{uk} = \emptyset$, then on condition that $\|Q(A)(B)\| = r$, the CA to $Q(A)(B)$ is S_t, and the SA to $Q(A)(B)$ is the proposition "$S = Y$", where Y is the specific value of S_t in a certain model. This proposition can often be re-expressed as a tripartite structure using the truth conditions of ordinary quantifiers. For instance, if $\|who(-)(B)\| = r$, then the CA to "$who(-)(B)$" is $(PERSON \cap B)_t$, i.e. those who are known to be persons belonging to B. Furthermore, suppose the value of $(PERSON \cap B)_t$ in a certain model is the singleton set $\{x\}$, i.e. x is the only person known to belong to B, then the SA to "$who(-)(B)$" is the proposition "$PERSON \cap B = \{x\}$", which can be re-expressed as the following tripartite structure "$(nobody\ except\ x)(-)(B)$".

Apart from "strongly exhaustive" questions requesting complete information concerning a subject matter, there are also "non-exhaustive" questions which request only partial information, as exemplified by the question "Who for example did John see". In this paper, WH-phrase "who for example" will be expressed as a non-exhaustive IQ "(*at least who*)". The resolvedness condition of this IQ can be written as

$$\|(at\ least\ who)(-)(B)\| = r \Leftrightarrow (PERSON \cap B)_t \neq \emptyset \vee (PERSON \cap B)_f = U \quad (7)$$

The condition above reflects the fact that the question "$(at\ least\ who)(-)(B)$" is resolved iff either one of the following situations holds: (1) at least one element is known to belong to $PERSON \cap B$; (2) all elements are known not to belong to $PERSON \cap B$. The resolvedness conditions of other non-exhaustive IQs can also be formulated as $S_t \neq \emptyset \vee S_f = U$ for an appropriate set S.

The CA to a non-exhaustive IQ is not unique and so I will provide the set of all possible CAs which can be determined as follows: let Q be a non-exhaustive IQ whose resolvedness condition has the form $S_t \neq \emptyset \vee S_f = U$, then on condition that $\|Q(A)(B)\| = r$, the CA set of $Q(A)(B)$ is

$$\text{CA set} = \begin{cases} \{X : \emptyset \neq X \subseteq S_t\}, & \text{if } S_t \neq \emptyset \\ \{\emptyset\}, & \text{if } S_t = \emptyset \end{cases} \quad (8)$$

For instance, if $\|(at\ least\ who)(-)(B)\| = r$, then the CA set of "$(at\ least\ who)(-)(B)$" is

$$\text{CA set} = \begin{cases} \{X : \emptyset \neq X \subseteq (PERSON \cap B)_t\}, & \text{if } (PERSON \cap B)_t \neq \emptyset \\ \{\emptyset\}, & \text{if } (PERSON \cap B)_t = \emptyset \end{cases} \quad (9)$$

The above piecewise-defined function provides the CA set under two situations. If $(PERSON \cap B)_t = \emptyset$, by (7) we must have $(PERSON \cap B)_f = U$, i.e. no person belongs to B and so the unique CA should be "nobody", represented by a singleton consisting of $\emptyset$. If $(PERSON \cap B)_t \neq \emptyset$, then every non-empty subset of $(PERSON \cap B)_t$, i.e. any set X satisfying $\emptyset \neq X \subseteq (PERSON \cap B)_t$, is an acceptable CA. So all these Xs are collected into a set, and the CA can be any member of this set.

To study inferences involving interrogatives, we need to define entailment and equivalence relations involving interrogatives. Under the present framework, it is straightforward to define these notions:

> Let $S = \{s_1, \ldots s_n\}$ be a set of questions / propositions and q a question, then S entails q (denoted $S \Rightarrow q$), iff in every model, $(\|s_1\| \in \{t, r\} \wedge \ldots \wedge \|s_n\| \in \{t, r\}) \Rightarrow \|q\| = r$.

(10)

> Let q_1 and q_2 be questions, then q_1 is equivalent to q_2 (denoted $q_1 \Leftrightarrow q_2$), iff in every model, $\|q_1\| = r \Leftrightarrow \|q_2\| = r$.

(11)

Based on the resolvedness conditions of IQs and the above definitions, we can derive valid inferential patterns of IQs. For example, it can be shown that the following equivalence, entailment and "interrogative syllogism" are all valid:

$$who(-)(\neg B) \Leftrightarrow (\textit{everybody except who})(-)(B) \tag{12}$$

$$who(-)(B) \Rightarrow (\textit{at least who})(-)(B) \tag{13}$$

$$\{which(M)(P), which(M)(S), S \subseteq M\} \Rightarrow which(S)(P) \tag{14}$$

Note that (14) above is a generalization of a result in [3]. An instance of this inference schema is that the two questions "Whom does Mary love" and "Who are the men" collectively entail the question "Which men does Mary love" (on the understanding that men are persons).

Moreover, we can also discuss the monotonicities of IQs, whose definitions are analogous to those of ordinary quantifiers (such as in [9]). It can be shown that the strongly exhaustive IQs considered in this paper are non-monotonic in all of their arguments, whereas the non-exhaustive IQs are increasing in all of their arguments within certain restricted domains. For example, it can be proved that within the domain $\{B : (PERSON \cap B)_f \neq U\}$, "$(\textit{at least who})(-)(B)$" is increasing in the argument B.

Finally, as pointed out by Nelken and Francez, the relation between a question and its SA can also be seen as an entailment relation. In this framework, we can prove the following:

$$\text{If } p \text{ is an SA to } q, \text{ then } p \Rightarrow q. \tag{15}$$

The above result shows that "$p \Rightarrow q$" is a necessary condition for "p is an SA to q". In other words, we can show that "p is not an SA to q" by showing that "$p \not\Rightarrow q$", thus providing us with a method to show that a certain proposition is not a resolved answer to a certain question. For instance, we can show that "(At least) John sang" is not a resolved SA to "Who sang", according to the strongly exhaustive interpretation of "who".

Keywords: interrogative quantifier, interrogative inference, bilattice, Generalized Quantifier Theory

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