Reconciling Inquisitive Semantics and Generalized Quantifier Theory*

Ka-fat CHOW

The Hong Kong Polytechnic University, Hong Kong
kfzhouy@yahoo.com

Abstract. This paper proposes a new treatment of quantifiers under the theoretical framework of Inquisitive Semantics (IS). After discussing the difficulty in treating quantifiers under the existing IS framework, I propose a new treatment of quantifiers that combines features of IS and the Generalized Quantifier Theory (GQT). My proposal comprises two main points: (i) assuming that the outputs of all quantifiers are non-inquisitive; and (ii) deriving a predicate X^* of type $s\to(e^n\to t)$ corresponding to each predicate X of type $e^n\to T$. By using X^*, we can then restore the traditional treatment of GQT under the IS framework. I next point out that to properly handle the pair list reading of some questions with "every", we have to revert to the old treatment of "every". I also introduce (and prove) a theorem that shows that the new treatment of "every" is just a special case of the old treatment, and conclude that the new treatment of all quantifiers other than "every" plus the old treatment of "every" is sufficient for the general purpose of treating quantified statements and questions.

Keywords: Inquisitive Semantics, Generalized Quantifier Theory, inquisitiveness, pair list reading.

1 Basic Notions of IS

In the 2010s, Inquisitive Semantics (IS) has risen to become an influential theory that provides a uniform treatment for declaratives and interrogatives. To facilitate subsequent discussion in this paper, I first introduce some basic notions of IS. Under IS, there are three tiers of notions that are based on possible worlds. The first tier consists of the possible worlds (hereinafter "worlds") themselves with type s. The second tier consists of information states (hereinafter "states"), which are sets of worlds, with type $s\to t$. The third tier consists of propositions, which are non-empty sets of states, i.e. sets of sets of worlds, with type $(s\to t)\to t$., that satisfy downward closure, i.e. whenever a state belongs to a proposition p, then all subsets of that state also belong to p. For convenience, the symbol T is often used as an abbreviation of the type $(s\to t)\to t$.

---

* This is the revised version of a paper in *The Proceedings of Logic and Engineering of Natural Language Semantics 15 (LENLS15)*. The revised paper was presented on 12 November 2018.

Let p be a proposition and let's assume that every proposition discussed in this paper consists of a finite number of states (which is a standard assumption in the IS literature). The alternatives of p are the maximal states of p, i.e. those states that are not proper subsets of other states. We say that p is informative iff[1] $\bigcup p \neq W$[2], where W represents the set of all worlds. We say that p is inquisitive iff p consists of more than one alternative. Apart from the usual set operations such as $\cup$ and $\cap$, there are also two special set operations under IS, namely relative pseudo-complement (represented by $\triangleright$) and absolute pseudo-complement (represented by ~), which can be defined as follows (in what follows, p and q are propositions, Power(S) represents the power set of the set S):

$$p \triangleright q = \{i \in \text{Power}(W): \text{Power}(i) \cap p \subseteq q\} \qquad (1)$$

$$\sim p = \text{Power}(W - \bigcup p) \qquad (2)$$

There are also two projection operators: the ! and ? operators, whose functions are to turn any proposition into an assertion (which is defined as a non-inquisitive proposition under IS) and a question (which is defined as a non-informative proposition under IS), respectively. These two operators can be defined as follows:

$$!p = \text{Power}(\bigcup p) \qquad (3)$$

$$?p = p \cup \sim p \qquad (4)$$

2 Treatment of Sub-sentential Constituents under IS

In recent years, attempts have been made under IS to treat sub-sentential constituents. The types of these constituents are all based on the type of propositions, i.e. T. For example, the types of unary and, in general, n-ary predicates are $e \rightarrow T$ and $e^n \rightarrow T$[3], respectively. Moreover, it is assumed under IS that all simple n-ary predicates (i.e. predicates with no internal structure) are non-inquisitive, i.e. the outputs of these functions are non-inquisitive propositions. For illustration, let's consider the following model.

Table 1. Model M1

$U^* = \{john, mary\}$, $W = \{w_1, w_2, w_3, w_4\}$

[1] In this paper, we use "iff" to represent "if and only if".

[2] In this paper, we use the symbols $\bigcup$ and $\bigcap$ to represent the generalized union and intersection operations, respectively.

[3] In this paper, I adopt the uncurried form of n-ary predicates, i.e. the input of an n-ary predicate is an n-tuple. Here I use e^n to represent the type of n-tuples of entities with type e.

sing =	john $\mapsto$[4] $\{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \varnothing\}$; mary $\mapsto$ $\{\{w_1, w_3\}, \{w_1\}, \{w_3\}, \varnothing\}$

One may check that the unary predicate "sing" given above is a function with type e→T. For each member x of U^*, this function maps x to the power set of the set of worlds in which "x sang" is true. Since this is the power set of a set, it contains only one alternative and is thus non-inquisitive. Now consider ?(sing(john)), which can be used to represent the question "Did John sing?". By using the definitions given above, one can calculate

$$?(\text{sing(john)}) = \{\{w_1, w_2\}, \{w_3, w_4\}, \{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}, \varnothing\} \quad (5)$$

Note that the above result does have the form of a proposition, i.e. a non-empty set of sets of worlds satisfying downward closure. Moreover, since $\cup$?(sing(john)) = W, this proposition is non-informative, i.e. a question. It has two alternatives, i.e. $\{w_1, w_2\}$, and $\{w_3, w_4\}$, which represent the two possible answers to the question "Did John sing?". For example, $\{w_1, w_2\}$ represents the answer "Yes" because w_1 and w_2 are exactly the worlds in which "John sang" is true under M1.

Quantifiers, an important subtype of sub-sentential constituents, are also treated in the recent IS literature. However, the treatment of quantifiers under IS as in [2-3, 9] is different from the traditional treatment under the Generalized Quantifier Theory (GQT). For example, the denotation of "every" is written in [2-3] as:

$$\text{every} = \lambda X\lambda Y[\cap_{x \in U^*}(X(x) \rhd Y(x))] \quad (6)$$

which looks quite different from that given in standard GQT literature (such as [7-8]):

$$\text{every} = \lambda X\lambda Y[X \subseteq Y] \quad (7)$$

Of course one may argue that the difference between (5) and (6) is superficial because the denotation in (5) is in fact a "translation" of the following first order statement into the IS language: $\forall x \in U^*[X(x) \rightarrow Y(x)]$, which is equivalent to the set theoretic statement $X \subseteq Y$. But not all quantified statements have equivalent first order statements. Consider the denotation of the quantifier "most":

$$\text{most} = \lambda X\lambda Y[|X \cap Y| / |X| > 1/2] \quad (8)$$

According to modern GQT studies (e.g. [8]), a quantified statement with "most" cannot be rewritten as a first order statement. Thus, it is not known under the existing IS framework how "most" should be treated. A consequence of this is that some quantifiers that have been successfully treated under GQT may not be treated in a comparably elegant way under the existing IS framework.

Moreover, there is also the issue of inquisitiveness of quantifiers. As will be shown in Section 4 below, the output of "every" is non-inquisitive if both of its arguments

4 The symbol $\mapsto$ here is used to represent the "maps to" relation between the input and output of a function.

are non-inquisitive, and is inquisitive if at least one argument is inquisitive. This property which looks quite complicated is useful for handling the "pair list" reading of some questions with "every".

What about the other quantifiers? As will be elaborated in more detail in Section 4, for constituent questions with quantifiers other than "every", there does not exist a reading similar to the "pair list" reading in which the quantifier takes a wider scope than the WH-word. Thus, for all quantifiers other than "every", we may assume a simpler property in terms of their inquisitiveness.

3 Proposed New Treatment of Quantifiers

3.1 The Proposal

To achieve a proper treatment of quantifiers under IS, I first assume that the outputs of all quantifiers (including "every" as long as we are not considering the pair list reading) are non-inquisitive. One advantage of this is that all quantifiers can be treated in a similar fashion. Another advantage is that quantified statements can be given a simple representation. In the existing IS framework, the output of the quantifier "someone" (as given in [9]) is inquisitive even when its argument is non-inquisitive. Thus, the assertion "Someone sang" has to be represented as !(someone(sing)). The ! operator here is necessary to ensure that the expression is non-inquisitive (i.e. an assertion). If the outputs of all quantifiers are non-inquisitive, then the aforesaid assertion can be represented more simply as someone(sing). Note that the aforesaid strategy is adequate for the usual purpose of treating quantified statements, unless we are considering the pair list reading or studying some special semantic-pragmatic aspects of some quantifiers, such as the study in [4].

I next observe that a simple n-ary predicate under IS, whose output is the power set of a set of worlds, in fact contains a lot of redundant information. For example, in the denotation of "sing" given in Model M1 (see Table 1), the output of sing(john) is $\{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \varnothing\}$, which contains redundant information because $\{w_1, w_2\}$ alone can tell us that John sang in w_1 and w_2. By eliminating the redundancy, we can derive predicates with a simpler type, i.e. $s\rightarrow(e^n\rightarrow t)$. More specifically, corresponding to each n-ary predicate X with type $e^n\rightarrow T$, there is a predicate X^* with type $s\rightarrow(e^n\rightarrow t)$ and the two predicates can be transformed to each other by the following formulae (in what follows, x and w are variables of types e^n and s, respectively):

$$X^* = \lambda w[\{x: \{w\} \in X(x)\}] \quad (9)$$

$$X = \lambda x[\text{Power}(\{w: x \in X^*(w)\})] \quad (10)$$

By using X^*, the traditional treatment of GQT can then be restored under the framework of IS. For example, the denotation of "every" under IS will become

$$\text{every} = \lambda X\lambda Y[\text{Power}(\{w: X^*(w) \subseteq Y^*(w)\})] \quad (11)$$

Since X^* and Y^* have type $s \to (e \to t)$ and w is a variable with type s, $X^*(w)$ and $Y^*(w)$ have type $e \to t$, which is the type of unary predicates under GQT, and so "$X^*(w) \subseteq Y^*(w)$" in (11) is exactly parallel to "$X \subseteq Y$" in (7).

In general, let Q be a monadic quantifier[5] under GQT with n unary predicates $X_1, \ldots X_n$ each of type $e \to t$ as arguments and $C(X_1, \ldots X_n)$ be the truth condition associated with Q, i.e. Q has the denotation $\lambda X_1 \ldots \lambda X_n[C(X_1, \ldots X_n)]$. Then there is a corresponding quantifier (also denoted Q) with n unary predicates (also denoted $X_1, \ldots X_n$) each of type $e \to T$ as arguments and the denotation of Q under IS is

$$\lambda X_1 \ldots \lambda X_n[\text{Power}(\{w: C(X_1^*(w), \ldots X_n^*(w))\})] \quad (12)$$

Note that according to (12) $Q(X_1)\ldots(X_n)$ is the power set of a set of worlds and is thus non-inquisitive because it contains only one alternative. This shows that the output of Q is non-inquisitive, which is consistent with the assumption above. By using (12), one can then write down the denotations of other quantifiers under IS. For example, the denotation of "most" under IS can be written as follows:

$$\text{most} = \lambda X \lambda Y[\text{Power}(\{w: |X^*(w) \cap Y^*(w)| / |X^*(w)| > 1/2\})] \quad (13)$$

The proper treatment of quantifiers can help extend the empirical coverage of IS, because in natural languages there are many questions containing quantifiers. Under IS, given a declarative proposition p, the corresponding polar question can be represented as "?p", where "?" is the projection operator defined in (4). Similarly, under IS a constituent question "Which X is Y?", where X and Y are unary predicates, can be represented as "which(X)(Y)", where "which" is a non-exhaustive interrogative operator defined as follows (the context sensitivity of "which" is ignored here)[6]:

$$\text{which} = \lambda X \lambda Y[?(\cup_{x \in U} (X \cap Y)(x))]^7 \quad (14)$$

For simplicity, only the "non-exhaustive" reading of interrogative operators is discussed in this paper. In brief, the non-exhaustive reading of the constituent question "Which X is Y?" only requires the answerer to provide at least one X that is Y or to answer that there is no X that is Y. The full list of X that is Y is not required. A discussion of the various "exhaustivity" of interrogative operators can be found in [9-10].

---

5 Monadic quantifiers are quantifiers all arguments of which are unary predicates. In case at least one argument is an n-ary predicate ($n > 1$), the quantifier is called polyadic.

6 Note that the following denotation of "which" is a bit different from those given in [3, 9] in that the following denotation includes a built-in ? operator. The inclusion of this operator is to ensure that "No X is Y" is an acceptable answer to the constituent question "Which X is Y?". In other words, I assume in this paper that "which" does not carry the existential presupposition.

7 For unary predicates X and Y and an y member x, $(X \cap Y)(x) = X(x) \cap Y(x)$.

3.2 Worked Examples

For illustration, let's consider the following model.

Table 2. Model M2[8]

U^* = {john, bill, mary, jane, katy}, W = {w_1, w_2, w_3}	
boy =	john $\mapsto$ Power(W); bill $\mapsto$ Power(W)
girl =	mary $\mapsto$ Power(W); jane $\mapsto$ Power(W); katy $\mapsto$ Power(W)
like =	(john, bill) $\mapsto$ {{w_1}, $\varnothing$}; (john, mary) $\mapsto$ {{w_2}, $\varnothing$}; (john, katy) $\mapsto$ {{w_2}, $\varnothing$}; (bill, jane) $\mapsto$ {{w_2, w_3}, {w_2}, {w_3}, $\varnothing$}; (bill, katy) $\mapsto$ {{w_3}, $\varnothing$}; (mary, jane) $\mapsto$ {{w_1, w_3}, {w_1}, {w_3}, $\varnothing$}; (mary, katy) $\mapsto$ {{w_1}, $\varnothing$}
boy^* =	w_1 $\mapsto$ {john, bill}; w_2 $\mapsto$ {john, bill}; w_3 $\mapsto$ {john, bill}
$girl^*$ =	w_1 $\mapsto$ {mary, jane, katy}; w_2 $\mapsto$ {mary, jane, katy}; w_3 $\mapsto$ {mary, jane, katy}
$like^*$ =	w_1 $\mapsto$ {(john, bill), (mary, jane), (mary, katy)}; w_2 $\mapsto$ {(john, mary), (john, katy), (bill, jane)}; w_3 $\mapsto$ {(bill, jane), (bill, katy), (mary, jane)}

To simplify presentation, I adopt the following convention: if the output of a function given a particular input is {$\varnothing$}, then that input (and output) will not be shown. Thus, it is understood that under M2, we have boy(mary) = {$\varnothing$} and like(john, john) = {$\varnothing$}. For convenience, I have also provided the denotations of boy^*, $girl^*$ and $like^*$ above. One may check that these results can be obtained by applying formula (9), and that the denotations of boy, girl and like can be obtained from these results by applying formula (10).

Now consider the polar question "Does some boy like most girls?". By using the ? operator and the standard GQT concepts for treating iterative quantifiers such as those in [6-8], this polar question can be formally represented as

$$?(\text{some}(\text{boy})(\text{most}(\text{girl})_{\text{ACC}}(\text{like}))) \tag{15}$$

---

[8] Note that the models M2 and M3 given in this paper are highly simplified models. They do not include all logically possible worlds (the total number of all such worlds is an astronomical number). For example, M2 does not include those worlds in which John is a girl and John likes herself. One may think that M2 and M3 are models that satisfy certain given preconditions. The satisfaction of these preconditions has greatly reduced the number of possible worlds in these two models.

where "ACC" represents the accusative case extension operator in [6] (note that "most girls" is in the accusative "semantic" case in the above polar question, hence the "ACC" operator). Let Q be a monadic quantifier. Then Q_{ACC} is an arity reducer that turns any binary predicate R to a unary predicate $Q_{ACC}(R)$ such that[9]

$$Q_{ACC}(R) = \lambda x[Q(\lambda y[R(x, y)])] \tag{16}$$

I next compute the denotation of (15) with respect to M2 step by step. To do this, I first use (16) to rewrite (15) as

$$?(some(boy)(\lambda x[most(girl)(\lambda y[like(x, y)])])) \tag{17}$$

I then calculate $\lambda y[like(x, y)]^*$ for each $x \in U'$. For example, for x = john, the most straightforward way to calculate $\lambda y[like(john, y)]^*$ is to make use of $like^*$, which tells us that John likes Bill in w_1, Mary and Katy in w_2 and nobody in w_3. So we have

$$\lambda y[like(john, y)]^* = w_1 \mapsto \{bill\};\ w_2 \mapsto \{mary, katy\};\ \ w_3 \mapsto \varnothing$$

Similarly, we can calculate

$$\lambda y[like(bill, y)]^* = w_1 \mapsto \varnothing;\ w_2 \mapsto \{jane\};\ \ w_3 \mapsto \{jane, katy\}$$

$$\lambda y[like(mary, y)]^* = w_1 \mapsto \{jane, katy\};\ w_2 \mapsto \varnothing;\ \ w_3 \mapsto \{jane\}$$

$$\lambda y[like(jane, y)]^* = w_1 \mapsto \varnothing;\ w_2 \mapsto \varnothing;\ w_3 \mapsto \varnothing$$

$$\lambda y[like(katy, y)]^* = w_1 \mapsto \varnothing;\ w_2 \mapsto \varnothing;\ w_3 \mapsto \varnothing$$

Using the denotations of most, $girl^*$ and $\lambda y[like(x, y)]^*$, I next calculate most(girl)(λy[like(x, y)]) for each $x \in U'$. For example, for x = john, among the three worlds, only $|girl^*(w_2) \cap \lambda y[like(john, y)]^*(w_2)| / |girl^*(w_2)| > 1/2$ is true, we thus have

$$most(girl)(\lambda y[like(john, y)]) = \{\{w_2\}, \varnothing\}$$

Similarly, we also have

$$most(girl)(\lambda y[like(bill, y)]) = \{\{w_3\}, \varnothing\}$$

$$most(girl)(\lambda y[like(mary, y)]) = \{\{w_1\}, \varnothing\}$$

$$most(girl)(\lambda y[like(jane, y)]) = \{\varnothing\}$$

$$most(girl)(\lambda y[like(katy, y)]) = \{\varnothing\}$$

Summarizing the above in the form of a unary predicate, we have

---

9 Set theoretic notation is used in [6]. In this paper, this notation is changed to λ-notation for consistency with the other parts of the paper.

$$\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])] = \begin{array}{l} \text{john} \mapsto \{\{w_2\}, \varnothing\}; \\ \text{bill} \mapsto \{\{w_3\},\varnothing\}; \\ \text{mary} \mapsto \{\{w_1\},\varnothing\}; \\ \text{jane} \mapsto \{\varnothing\}; \\ \text{katy} \mapsto \{\varnothing\} \end{array}$$

Transforming the above predicate into the corresponding starred version by using formula (9), we have:

$$\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]^* = w_1 \mapsto \{\text{mary}\}; w_2 \mapsto \{\text{john}\}; w_3 \mapsto \{\text{bill}\} \qquad (18)$$

Using the denotations of some, boy^* and $\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]^*$, I then calculate

$$\text{some}(\text{boy})(\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]) = \{\{w_2, w_3\}, \{w_2\}, \{w_3\}, \varnothing\} \qquad (19)$$

Finally, using the definition of ?, I can then calculate

$$?(\text{some}(\text{boy})(\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])) = \{\{w_2, w_3\}, \{w_1\}, \{w_2\}, \{w_3\}, \varnothing\} \quad (20)$$

The final result above contains two alternatives corresponding to the two answers to the polar question "Does some boy like most girls?" under M2, namely $\{w_2, w_3\}$ corresponding to "Yes" and $\{w_1\}$ corresponding to "No", because it is true in w_2 and w_3 (but not w_1) that some boy likes most girls.

Next consider the constituent question "Which boy likes most girls?". By using the interrogative operator "which", this constituent question can be formally represented as

$$\text{which}(\text{boy})(\text{most}(\text{girl})_{\text{ACC}}(\text{like})) \qquad (21)$$

I next compute the denotation of the above with respect to M2. As in the above example, I first use (16) to rewrite the above as

$$\text{which}(\text{boy})(\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]) \qquad (22)$$

As I have already calculated the denotation of $\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]$ above, what I have to do next is to use the denotations of which, boy and $\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]$ to calculate the denotation of (22). To do this, I first calculate $(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(z)$ for every $z \in U^*$:

$$(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(\text{john}) = \{\{w_2\}, \varnothing\}$$

$$(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(\text{bill}) = \{\{w_3\}, \varnothing\}$$

$$(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(\text{mary}) = \{\varnothing\}$$

$$(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(\text{jane}) = \{\varnothing\}$$

$$(\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])])(\text{katy}) = \{\varnothing\}$$

From the above, we have

$$\bigcup_{z \in U'} (\text{boy} \cap \lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]))(z) = \{\{w_2\}, \{w_3\}, \varnothing\} \quad (23)$$

And finally we obtain the result

$$\text{which}(\text{boy})(\lambda x[\text{most}(\text{girl})(\lambda y[\text{like}(x, y)])]) = \{\{w_2\}, \{w_3\}, \{w_1\}, \varnothing\} \quad (24)$$

The final result above contains three alternatives corresponding to the three answers to the constituent question "Which boy likes most girls?" under M2, namely $\{w_2\}$ corresponding to "John", $\{w_3\}$ corresponding to "Bill" and $\{w_1\}$ corresponding to "No boy", because it is precisely John and precisely Bill who likes most girls in w_2 and w_3 respectively, whereas no boy likes most girls in w_1.

4 Pair List Reading

4.1 The Phenomenon

However, the new treatment of quantifiers proposed in this paper cannot handle the pair list reading of some questions. Consider the question "Which book did every girl read?", which is ambiguous between at least two readings: the "individual reading" and the "pair list reading". Under the individual reading, the question can be paraphrased as "Which book x is such that every girl read x?", and can thus be formally represented as

$$\text{which}(\text{book})(\text{every}(\text{girl})_{\text{NOM}}(\text{read})) \quad (25)$$

where "NOM" represents the nominative case extension operator in [6] (note that "every girl" is in the nominative "semantic" case in the above question, hence the "NOM" operator). The individual reading can be handled by the concepts and method discussed in the previous section, except that we further need the following definition of the "NOM" operator:

$$Q_{\text{NOM}}(R) = \lambda y[Q(\lambda x[R(x, y)])] \quad (26)$$

The individual reading will not be further discussed. What I am interested in here is the pair list reading, which can be paraphrased as "For every girl x, which book did x read?", and can thus be formally represented as[10]

$$\text{every}(\text{girl})(\text{which}(\text{book})_{\text{ACC}}(\text{read})) \quad (27)$$

Under the pair list reading, "every" takes a wider scope than "which" (whereas "every" takes a narrower scope than "which" in (25)). Note that if we use the new treatment of "every" as given in (11) to handle (27), we have to transform $\text{which}(\text{book})_{\text{ACC}}(\text{read})$ into the starred version by using (9). But since this is a question

---

[10] Here "which(book)" is treated as a quantifier. Note that "which men", "how many students" and the like are called "interrogative quantifiers" in [2].

and is thus non-informative, we would then have $\text{which(book)}_{ACC}(\text{read})^*(w) = U'$ for all w. But then we would have $\text{girl}^*(w) \subseteq \text{which(book)}_{ACC}(\text{read})^*(w)$ for all w and hence $\text{every(girl)(which(book)}_{ACC}\text{(read))} = \text{Power}(W)$, which is obviously an incorrect result. What can we do?

To properly handle the pair list reading, we have to revert to the old treatment of "every" given in (6). But there is now a question that needs to be addressed. Now that we have two treatments of "every", i.e. the old treatment given in (6) and the new treatment given in (11), we have to make sure that (6) and (11) are consistent with each other. This is guaranteed by the following theorem (the proof of which will be given in Subsection 4.3):

Theorem 1: Let X and Y be non-inquisitive unary predicates. Then $\text{Power}(\{w: X^*(w) \subseteq Y^*(w)\}) = \bigcap_{x \in U'}(X(x) \rhd Y(x))$.

By comparing the right hand sides of (6) and (11), one can see that (6) is reduced to (11) when X and Y, i.e. the two arguments of "every", are both non-inquisitive by virtue of this theorem, and so the new treatment of "every" is in fact a special case of the old treatment. When its two arguments are both non-inquisitive, one can use the reduced form (11) for convenience.

But then we have a further question: do we need to do the same for other quantifiers as we did for "every" above? The fact is that for other quantifiers, there is no similar scope ambiguity between the quantifier and a WH-word as in the case of "every". Consider the question "Which book is recommended by some teacher?" which contains "some". Apart from the individual reading in which "some" takes a narrower scope than "which", i.e. a reading which can be paraphrased as "Which book x is such that some teacher recommends x?", does this question also have a reading in which "some" takes a wider scope than "which", i.e. a reading which can be paraphrased as "Name some teacher x and tell me which book x recommends"? In the literature, such a reading is called the "choice reading". According to many scholars (including [1]), "choice reading" questions do not exist in natural languages. For other quantifiers, it is even less likely that they would give rise to a reading in which the quantifier takes a wider scope than a WH-word. This means that we do not need to invoke the old treatment of these quantifiers as in the case of "every".

In conclusion, the new treatment of all quantifiers other than "every" as proposed in this paper plus the old treatment of "every" (which in fact includes the new treatment of "every" as a special case) is sufficient for the general purpose of treating quantified statements and questions.

4.2 A Worked Example

In this subsection, I will illustrate the computation of the pair list reading. Consider the following model.

Table 3. Model M3

$U^* = \{$john, mary, jane, RC, OT, DC[11]$\}$, $W = \{w_1, w_2, w_3\}$	
boy =	john $\mapsto$ Power(W)
girl =	mary $\mapsto$ Power(W); jane $\mapsto$ Power(W)
book =	RC $\mapsto$ Power(W); OT $\mapsto$ Power(W); DC $\mapsto$ Power(W)
read =	(john, RC) $\mapsto \{\{w_1\}, \varnothing\}$; (john, OT) $\mapsto \{\{w_2, w_3\}, \{w_2\}, \{w_3\}, \varnothing\}$; (mary, RC) $\mapsto \{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \varnothing\}$; (mary, OT) $\mapsto \{\{w_1\}, \varnothing\}$; (mary, DC) $\mapsto \{\{w_3\}, \varnothing\}$; (jane,RC) $\mapsto \{\{w_2\}, \varnothing\}$; (jane, OT) $\mapsto \{\{w_1, w_2\}, \{w_1\}, \{w_2\}, \varnothing\}$; (jane, DC) $\mapsto \{\{w_3\}, \varnothing\}$
boy^* =	$w_1 \mapsto$ {john}; $w_2 \mapsto$ {john}; $w_3 \mapsto$ {john}
girl^* =	$w_1 \mapsto$ {mary, jane}; $w_2 \mapsto$ {mary, jane}; $w_3 \mapsto$ {mary, jane}
book^* =	$w_1 \mapsto$ {RC, OT, DC}; $w_2 \mapsto$ {RC, OT, DC}; $w_3 \mapsto$ {RC, OT, DC}
read^* =	$w_1 \mapsto$ {(john, RC), (mary, RC), (mary, OT), (jane, OT)}; $w_2 \mapsto$ {(john, OT), (mary, RC), (jane, RC), (jane, OT)}; $w_3 \mapsto$ {(john, OT), (mary, DC), (jane, DC))}

I next compute the denotation of (27), i.e. the pair list reading of "Which book did every girl read?", with respect to M3. To do this, I first use (16) to rewrite (27) as

$$\text{every(girl)}(\lambda x[\text{which(book)}(\lambda y[\text{read}(x, y)])]) \qquad (28)$$

I then calculate which(book)(λy[read(x, y)]) for each $x \in U^*$. For example, for x = john, since λy[read(john, y)] = RC $\mapsto \{\{w_1\}, \varnothing\}$; OT $\mapsto \{\{w_2, w_3\}, \{w_2\}, \{w_3\}, \varnothing\}$, by (14), we have

$$\text{which(book)}(\lambda y[\text{read(john, y)}]) = \{\{w_1\}, \{w_2, w_3\}, \{w_2\}, \{w_3\}, \varnothing\}$$

Similarly, we also have

$$\text{which(book)}(\lambda y[\text{read(mary, y)}]) = \{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$$

$$\text{which(book)}(\lambda y[\text{read(jane, y)}]) = \{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$$

$$\text{which(book)}(\lambda y[\text{read(RC, y)}]) = \text{Power(W)}$$

$$\text{which(book)}(\lambda y[\text{read(OT, y)}]) = \text{Power(W)}$$

$$\text{which(book)}(\lambda y[\text{read(DC, y)}]) = \text{Power(W)}$$

[11] RC, OT and DC can be seen as abbreviations of *Robinson Crusoe*, *Oliver Twist* and *David Copperfield*, respectively.

Summarizing the above in the form of a unary predicate, we have

$$\lambda x[\text{which}(\text{book})(\lambda y[\text{read}(x, y)])] = \begin{array}{l} \text{john} \mapsto \{\{w_1\}, \{w_2, w_3\}, \{w_2\}, \{w_3\}, \varnothing\}; \\ \text{mary} \mapsto \{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}; \\ \text{jane} \mapsto \{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}; \\ \text{RC} \mapsto \text{Power}(W); \\ \text{OT} \mapsto \text{Power}(W); \\ \text{DC} \mapsto \text{Power}(W) \end{array}$$

Finally, to compute (28), I use (6) and (1) to rewrite (28) as

$$\bigcap_{z \in U'}(\{i \in \text{Power}(W): \text{Power}(i) \cap \text{girl}(z) \subseteq \lambda x[\text{which}(\text{book})(\lambda y[\text{read}(x, y)])](z)\}) \quad (29)$$

To compute the above formula, I first have to find out all sets of worlds i such that Power(i) $\cap$ girl(z) $\subseteq$ λx[which(book)(λy[read(x, y)])](z) for each z $\in$ U'. For example, in case z = john, since girl(john) = $\{\varnothing\}$, Power(i) $\cap$ girl(john) must be a subset of λx[which(book)(λy[read(x, y)])](john) for any i, and so the required set of sets of worlds in this case is Power(W). Similarly, in case z = RC, OT or DC, the required set of sets of worlds is also Power(W).

In case z = mary, since girl(mary) = Power(W) and λx[which(book)(λy[read(x, y)])](mary) = $\{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$, in order for Power(i) $\cap$ girl(mary) to be a subset of $\{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$, i must be a member of $\{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$ and every such member satisfies the requirement. Thus, the required set of sets of worlds in this case is $\{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$. Similarly, in case z = jane, the required set of sets of worlds is also $\{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\}$.

I then find the intersection of all the above sets of sets of world and finally obtain

$$\text{every}(\text{girl})(\text{which}(\text{book})_{\text{ACC}}(\text{read})) = \{\{w_1, w_2\}, \{w_3\}, \{w_1\}, \{w_2\}, \varnothing\} \quad (30)$$

The final result above contains two alternatives corresponding to the two answers to the pair list reading of the question "Which book did every girl read?" under M3, namely $\{w_1, w_2\}$ corresponding to "Mary read RC and Jane read OT", and $\{w_3\}$ corresponding to "Both Mary and Jane read DC". Note that although the books that Mary and Jane precisely read in w_1 and w_2 are not the same (Mary also read OT in w_1 while Jane also read RC in w_2), w_1 and w_2 are grouped under the same alternative in (30) because "which" in this question has a non-exhaustive reading, i.e. "Mary read RC and Jane read OT" is an acceptable answer to the question in both w_1 and w_2.

4.3 Some Proofs

In this subsection, I will prove Theorem 1. But before doing this, I have to prove three lemmas first.

Lemma 2: Let p(w, x) be a proposition with variables w and x. Then Power($\{w: \forall x \in U' [p(w, x)]\}$) = $\bigcap_{x \in U'}$(Power($\{w: p(w, x)\}$)).

Proof: Let V be an arbitrary set of worlds. Then

$V \in \text{Power}(\{w: \forall x \in U' [p(w, x)]\})$

iff $V \subseteq \{w: \forall x \in U' [p(w, x)]\}$

iff $\forall w \in V\ \forall x \in U' [p(w, x)]$

iff $\forall x \in U'\ \forall w \in V [p(w, x)]$

iff $\forall x \in U' [V \subseteq \{w: p(w, x)\}]$

iff $\forall x \in U' [V \in \text{Power}(\{w: p(w, x)\})]$

iff $V \in \bigcap_{x \in U'} (\text{Power}(\{w: p(w, x)\}))$

From the above, we have $\text{Power}(\{w: \forall x \in U' [p(w, x)]\}) = \bigcap_{x \in U'} (\text{Power}(\{w: p(w, x)\}))$. □

Lemma 3: Let i, s and t be sets of worlds. Then $i \cap s \subseteq t$ iff $\text{Power}(i) \cap \text{Power}(s) \subseteq \text{Power}(t)$.

Proof: (i) First assume that $i \cap s \subseteq t$. Let j be an arbitrary set of worlds and $j \in \text{Power}(i) \cap \text{Power}(s)$, i.e. $j \in \text{Power}(i) \wedge j \in \text{Power}(s)$. But this is equivalent to $j \subseteq i \wedge j \subseteq s$, i.e. $j \subseteq i \cap s$. From this we have $j \subseteq t$, i.e. $j \in \text{Power}(t)$. We have thus proved that $\forall j[j \in \text{Power}(i) \cap \text{Power}(s) \rightarrow j \in \text{Power}(t)]$, i.e. $\text{Power}(i) \cap \text{Power}(s) \subseteq \text{Power}(t)$.
(ii) Next assume that $\text{Power}(i) \cap \text{Power}(s) \subseteq \text{Power}(t)$. Let w be an arbitrary world and $w \in i \cap s$, i.e. $w \in i \wedge w \in s$. But this is equivalent to $\{w\} \in \text{Power}(i) \wedge \{w\} \in \text{Power}(s)$, i.e. $\{w\} \in \text{Power}(i) \cap \text{Power}(s)$. From this we have $\{w\} \in \text{Power}(t)$, i.e. $w \in t$. We have thus proved that $\forall w[w \in i \cap s \rightarrow w \in t]$, i.e. $i \cap s \subseteq t$.
Combining (i) and (ii) above, the lemma is proved. □

Lemma 4: Let p and q be non-inquisitive propositions. Then $p \triangleright q = \text{Power}(\{w: \{w\} \in p \rightarrow \{w\} \in q\})$.

Proof: Since p and q are non-inquisitive propositions, by the definition of inquisitiveness, each of p and q has exactly one alternative, say s and t, respectively. By the definition of alternatives, we have $p = \text{Power}(s)$ and $q = \text{Power}(t)$. From this we have

	$\text{Power}(\{w: \{w\} \in p \rightarrow \{w\} \in q\})$	
=	$\{i: i \subseteq \{w: \{w\} \in p \rightarrow \{w\} \in q\}\}$	
=	$\{i: i \subseteq \{w: \{w\} \in \text{Power}(s) \rightarrow \{w\} \in \text{Power}(t)\}\}$	
=	$\{i: i \subseteq \{w: \{w\} \subseteq s \rightarrow \{w\} \subseteq t\}\}$	
=	$\{i: i \subseteq \{w: w \in s \rightarrow w \in t\}\}$	
=	$\{i: \forall v \in W[v \in i \rightarrow v \in \{w: w \in s \rightarrow w \in t\}]\}$	
=	$\{i: \forall v \in W[(v \in i \wedge v \in s) \rightarrow v \in t]\}$	
=	$\{i: i \cap s \subseteq t\}$	
=	$\{i: \text{Power}(i) \cap \text{Power}(s) \subseteq \text{Power}(t)\}$	by lemma 3
=	$\{i: \text{Power}(i) \cap p \subseteq q\}$	
=	$p \triangleright q$	by (1)

□

Proof of Theorem 1: Let X and Y be non-inquisitive unary predicates and z be an arbitrary variable of type e. Then X(z) and Y(z) are non-inquisitive propositions.

$$
\begin{aligned}
& \text{Power}(\{w: X^*(w) \subseteq Y^*(w)\}) \\
= \quad & \text{Power}(\{w: \{x: \{w\} \in X(x)\} \subseteq \{x: \{w\} \in Y(x)\}\}) && \text{by (9)} \\
= \quad & \text{Power}(\{w: \forall z \in U' [z \in \{x: \{w\} \in X(x)\} \to z \in \{x: \\
& \{w\} \in Y(x)\}]\}) \\
= \quad & \text{Power}(\{w: \forall z \in U' [\{w\} \in X(z) \to \{w\} \in Y(z)]\}) \\
= \quad & \cap_{z \in U'} (\text{Power}(\{w: \{w\} \in X(z) \to \{w\} \in Y(z)\})) && \text{by Lemma 2} \\
= \quad & \cap_{z \in U'} (X(z) \rhd Y(z)) && \text{by Lemma 4}
\end{aligned}
$$

□

5 Conclusion

In this paper, I have proposed a new treatment of quantifiers. By combining features of IS and GQT, this new treatment is able to extend the coverage of IS to questions with quantifiers as well as retain the traditional truth conditions of quantifiers under GQT. I have also pointed out that the old treatment of "every" is still needed for treating the pair list reading of some questions with "every". But apart from this, the new treatment of all other quantifiers is sufficient for the general purpose of treating quantified statements and questions. In fact, even the new treatment of "every" is useful and convenient in many cases, provided that we are not treating the pair list reading. I have also shown that the new treatment of "every" is just a special case of the old treatment.

However, given the limited space, this paper has only discussed the basics of a theory of quantified statements and questions that combines IS and GQT. More specifically, regarding quantifiers, this paper has only discussed monadic quantifiers and iteration of these quantifiers. Regarding interrogatives, this paper has only discussed polar questions and constituent questions with the non-exhaustive "which". In future studies, the coverage of this theory can be extended to non-iterated polyadic quantifiers (such as those discussed in [7-8]) and other types of questions (such as the alternative questions, open disjunctive questions, rising interrogatives and tag questions discussed in [1, 5]) as well as constituent questions of other types of exhaustivity (such as the strongly exhaustive and weakly exhaustive readings discussed in [9-10]).

References

1. Ciardelli, I., Groenendijk, J., Roelofsen, F.: Inquisitive Semantics textbook manuscript, downloadable at https://semanticsarchive.net/Archive/WFjYTUwN/book.pdf, last accessed 2018/10/20.
2. Ciardelli, I., Roelofsen, F.: An inquisitive perspective on modals and quantifiers. Annual Review of Linguistics 4, 129-149 (2018).
3. Ciardelli, I., Roelofsen, F., Theiler, N.: Composing alternatives. Linguistics and Philosophy 40(1), 1-36 (2017).
4. Coppock, E., Brochhagen, T.: Raising and resolving issues with scalar modifiers. Semantics and Pragmatics 6(3), 1-57 (2013).

5. Farkas, D.F., Roelofsen, F.: Division of labor in the interpretation of declaratives and interrogatives. Journal of Semantics 34, 237-289 (2017).
6. Keenan, E.L: Semantic case theory. In: Groenendijk, J. et al (eds.) Proceedings of the Sixth Amsterdam Colloquium. ITLI (1987).
7. Keenan, E.L., D. Westerståhl.: Generalized quantifiers in linguistics and logic. In van Benthem, J., ter Meulen, A. (eds.) Handbook of Logic and Language (Second edition). Elsevier Science, Amsterdam, 859-910 (2011).
8. Peters, S., Westerståhl, D.: Quantifiers in Language and Logic. Clarendon Press, Oxford (2006).
9. Theiler, N.: A multitude of answers: embedded questions in typed inquisitive semantics, M.Sc. thesis, Universiteit van Amsterdam (2014).
10. Theiler, N., Roelofsen, F., Aloni, M., A uniform semantics for declarative and interrogative complements. Journal of Semantics (2018).