A Semantic Model for Interrogatives based on Generalized Quantifiers and Bilattices*

Ka-fat Chow

The Hong Kong Polytechnic University

kfzhouy@yahoo.com

Abstract. In this paper, I will develop a semantic model for interrogatives, an important sentence type expressing a special aspect of uncertainty. The model is based on the notions of generalized quantifiers and bilattices, and is used to model several aspects of interrogative semantics, including resolvedness conditions, answerhood, exhaustivity and interrogative inferences. It will be shown that the semantic model satisfies a number of adequacy criteria.

Keywords. interrogatives, generalized quantifiers, bilattices, resolvedness conditions, answerhood, interrogative inferences

1 Introduction

Interrogatives are an important sentence type in natural language expressing a special aspect of uncertainty. Yet the study on interrogatives in logic and formal semantics has been a difficult task because classical logical and formal semantics are basically truth conditional while there is not an intuitive and uncontroversial notion of truth values for interrogatives. While the topic of interrogatives seems to be a linguistic one, some scholars (such as [2], [6]) have studied this topic from the perspectives of logic and formal semantics and have identified a number of aspects for study. The aspects studied in this paper, i.e. resolvedness, exhaustivity, answerhood, interrogative inferences, are the standard ones in the studies on interrogatives. A good summary of these aspects can be found in [6].

According to [11], an adequate semantic model for interrogatives should satisfy the following adequacy criteria: 1. material adequacy – semantic notions of answerhood, entailment and equivalence should be definable under the model; 2. formal adequacy – the semantic notions should be interpretable as set-theoretic relations / operations; 3. empirical adequacy – the semantic notions should correspond to native speaker intuitions. What this criterion in fact meant according to [11] is that certain inferential relations that are intuitively correct should be provable under the model.

* This is the author-final version of a paper in D. Grossi, O. Roy, and H. Huang (Eds.): *Logic, Rationality and Interaction, Lecture Notes in Computer Science*, Volume 8196/2013, pp. 54–67, 2013. The final publication is available at www.springerlink.com.

In this paper, I will propose a semantic model that satisfies the aforesaid adequacy criteria. This model combines elements from a framework developed by [7, 8] that is based on generalized quantifiers (GQs) and a framework developed by [10, 11] that is based on bilattices. In Section 2, I will review the basic ideas proposed in [7, 8] and [10, 11], and point out some of their merits and demerits, and the need for enhancing and combining the frameworks. In Section 3, I will introduce the enhanced model and discuss the formal semantics of various types of interrogatives. In Section 4, I will discuss the issue of interrogative inferences. In Section 5, I will conclude the paper by discussing how the adequacy criteria are satisfied by my model.

2 Two Previous Models on Interrogatives

According to [5], the semantic frameworks for interrogatives may be classified into two broad approaches – the Categorial Approach and the Propositional Approach. Under the Categorial Approach, an interrogative is seen as an incomplete object, i.e. a function, which requires a constituent answer for completion. Since different constituent answers correspond to different semantic types, this approach does not assume a uniform type for interrogatives.

The semantic framework developed by [7, 8] is an example of the Categorial Approach. This framework is based on the Generalized Quantifier Theory (GQT)[1] and views WH-words as a special type of GQs, i.e. interrogative quantifiers (IQs). In a nutshell, a GQ can be seen as a second-order predicate with ordinary sets as arguments. The semantics of a GQ can be delineated by its truth condition expressed as a set-theoretic relation. For example, *every* can be seen as a GQ with 2 sets as arguments satisfying the truth condition $\|every(\mathrm{A}, \mathrm{B})\| = \mathrm{t} \Leftrightarrow \mathrm{A} \subseteq \mathrm{B}$ [2]. Different from ordinary GQs, IQs have an additional argument corresponding to the answer to the interrogative. The purpose of this answer argument is to make the interrogaive become a proposition. For example, under this framework, the truth condition of *which* can be represented by $\|which(\mathrm{A}, \mathrm{B}, \mathrm{X})\| = \mathrm{t} \Leftrightarrow \mathrm{A} \cap \mathrm{B} = \mathrm{X}$ [3], where X is the answer argument. This truth condition says that ‘X’ is the answer to the interrogative “Which ‘A’ is ‘B’?” iff $\mathrm{A} \cap \mathrm{B} = \mathrm{X}$ [4].

---

1 This paper adopts the standard notation as used in [12] for denoting quantified statements. Using this notation, a quantified statement such as “Every boy sang” is represented by *every*(BOY, SING), where *every* (in italics) is a quantifier with BOY and SING as arguments. Here the sets BOY and SING are semantic denotations of “boy” and “sang”, respectively.

2 In this paper, I use $\|\mathrm{s}\|$ to denote the truth value of a proposition / question s and “t” to denote the truth value “true”.

3 Strictly speaking, according to [7, 8], the truth condition of *which* should involve a context set because the interpretation of *which* is dependent on context. For simplicity, I have ignored the context-dependent effect of *which* in this paper.

4 In this paper, I use ‘S’ to denote the natural language word / phrase corresponding to the set S. I also use ‘s’ to denote the natural language declarative / interrogative sentence corresponding to the proposition / question s.

Contrary to the Categorial Approach, the Propositional Approach assumes that interrogatives are of one uniform type and the semantic type of interrogatives is to be analysed in terms of propositions.

The semantic framework developed by [10, 11] is an example of the Propositional Approach. This framework adopts the language of First Order Logic augmented by the symbol "?" for forming questions. Semantically, this framework adopts a bilattice model. According to [1], a bilattice is an algebraic structure composed of two complete lattices ordered separately but sharing a common negation operator "¬", such that "¬" reverses the order in one constituent lattice but preserves the order in the other. Now the framework developed by [10, 11] assumes a uniform type for both declarative and interrogative sentences. The denotation of declaratives and interrogatives are thus both truth values. However, to distinguish declaratives and interrogatives, they distinguish 2 subsets of truth values. For declaratives, there are 3 truth values: t ("known to be true"), f ("known to be false") and uk ("unknown whether true or false"). For interrogatives, they borrow the concept of "resolvedness" from [4] and assume 2 truth values: r ("resolved") and ur ("unresolved"). These 5 truth values thus form a bilattice composed of a declarative lattice and an interrogative lattice. The declarative lattice is ordered by f ≤ uk ≤ t and the interrogative lattice ordered by ur ≤ r. Obviously, "¬" reverses the order in the declarative lattice because if p_1 and p_2 are propositions and $\|p_1\| \leq \|p_2\|$, then $\|\neg p_2\| \leq \|\neg p_1\|$. As for the negation of interrogatives, the discussion will be postponed to Section 4.

Under this framework, the semantics of interrogatives is expressed by the resolvedness conditions which relate the two groups of truth values. For illustration, consider the polar interrogative "Did Mary kiss John?". The formal representation and resolvedness condition of this interrogative is $\|?(\text{KISS}(m, j))\| = r$ iff $\|\text{KISS}(m, j)\| \in \{t, f\}$, which means this polar interrogative is resolved iff it is known whether Mary kissed John.

Compared with the GQT framework developed by [7, 8], the bilattice framework developed by [10, 11] has some merits. Since their framework has a clear definition for truth values of both declaratives and interrogatives, it is straightforward to define entailment and equivalence relations between interrogatives and is thus convenient to study the issue of interrogative inferences under this model. On the other hand, interrogatives are interpreted as functions under the GQT framework. Although we may define entailment as set inclusion (note that functions can be seen as sets), this definition is only applicable to objects of the same category. Since interrogatives may belong to different categories, it is not clear how to come up with an appropriate definition for the general entailment relation between interrogatives.

Nevertheless, the bilattice framework developed by [10, 11] can only deal with a very small set of WH-words because it uses only one operator "?x" for WH-interrogatives. This is in sharp contrast with the GQT framework developed by [7, 8] which has defined a whole range of IQs for different WH-words. Thus, in comparison with the bilattice framework, the GQT framework has greater expressive power. It is also an attractive model because WH-words do share certain characteristics with ordinary GQs. In fact, in the GQT literature, WH-words are sometimes seen as a subtype of quantifiers. Moreover, it is also found that IQs possess certain properties that are thoroughly studied in GQT, such as conservativity, monotonicity, intersectivity, etc.

Given the merits and demerits of the aforesaid two frameworks, I will propose a novel semantic model for interrogatives that combines the merits and avoids the demerits of the two frameworks. Moreover, this semantic model will also deal with certain phenomena that are not dealt with in the two frameworks, such as non-exhaustive interrogatives and certain types of interrogative inferences.

3 A Novel Semantic Model for Interrogatives

3.1 Strongly Exhaustive IQs

The novel semantic model will adopt a bilattice structure that distinguishes 5 truth values as described in Section 2. Apart from this, we need some additional definitions. Under a 2-valued universe, with respect to every concept we have two complementary sets, e.g. X and ¬X. But under a 3-valued universe, we need 3 notions: X_t, X_f and X_{uk}, with the following definitions (in what follows, U denotes the universe or domain of discourse):

$$X_t = \{x \in U: \| x \in X \| = t\} \tag{1}$$

$$X_f = \{x \in U: \| x \in X \| = f\} \tag{2}$$

$$X_{uk} = \{x \in U: \| x \in X \| = uk\} \tag{3}$$

Thus, X_t, X_f and X_{uk} are sets containing elements that are known to belong to X, known not to belong to X and unknown whether to belong to X, respectively.

We now consider WH-interrogatives. Following [7, 8], I will treat these interrogatives as quantified statements containing IQs. But contrary to [7, 8], I do not employ the notion of "answer arguments" and will treat IQs in the same way as ordinary GQs. For instance, since in everyday use, "which" is used with a noun phrase and a verb phrase, such as in "Which boy sang?", *which* will be treated as an IQ with 2 arguments, just like the ordinary GQ *every*. Thus, the WH-interrogative "Which boy sang?" will be expressed as *which*(BOY, SING). In this way, IQs are similar to ordinary GQs as they function as second-order predicates with ordinary sets as arguments. Moreover, just like ordinary GQs, the semantics of IQs will be delineated by their truth conditions (or more precisely, resolvedness conditions) expressed as set-theoretic relations.

How can we derive the resolvedness condition of an IQ like *which*? Before answering this question, I need to introduce the notion of "exhaustivity", which is concerned with what constitutes an acceptable answer to a certain interrogative. From the literature, we can identify two most important types of exhaustivity: strong exhaustivity and non-exhaustivity. While non-exhaustivity only requires the answer to contain some true and no false information requested by the interrogative, strong exhaustivity requires the answer to contain all and only (i.e. exactly) the true information. In other words, strongly exhaustive answers differ from non-exhaustive ones in that the former are unique while the latter are not. For example, if it is known that John and Bill are exactly the boys who sang, then "John and Bill and no other boys" would be a strongly exhaustive answer to the interrogative "Which boy sang?". Since strong exhaustivity is easier to handle and is assumed by the most important theories on interrogatives, including [7, 8] and [10, 11], the simplest IQs are interpreted as strongly exhaustive IQs in this paper.

Under the strongly exhaustive interpretation, we know the answer to the interrogative "Which boy sang?" iff for every element x of U, we know whether x is a boy who sang. In other words, there is no element x such that we do not know whether x is a boy who sang. Thus, the resolvedness condition of "Which boy sang?" can be written as $\| \mathit{which}(\mathrm{BOY}, \mathrm{SING}) \| = \mathrm{r} \Leftrightarrow (\mathrm{BOY} \cap \mathrm{SING})_{\mathrm{uk}} = \varnothing$. This condition reflects the following intuition: $(\mathrm{BOY} \cap \mathrm{SING})_{\mathrm{uk}}$ represents the area of uncertainty with respect to the interrogative "Which boy sang?". If this area is empty, then the uncertainty does not exist and the interrogative is thus resolved.

The resolvedness conditions of *which* and some other commonly used strongly exhaustive IQs can be generalized as follows[5]:

$$\| \mathit{which}(\mathrm{A}, \mathrm{B}) \| = \mathrm{r} \Leftrightarrow (\mathrm{A} \cap \mathrm{B})_{\mathrm{uk}} = \varnothing \tag{4}$$

$$\| (\mathit{all\ except\ which})(\mathrm{A}, \mathrm{B}) \| = \mathrm{r} \Leftrightarrow (\mathrm{A} - \mathrm{B})_{\mathrm{uk}} = \varnothing \tag{5}$$

$$\| \mathit{who}(\mathrm{B}) \| = \mathrm{r} \Leftrightarrow (\mathrm{PERSON} \cap \mathrm{B})_{\mathrm{uk}} = \varnothing \tag{6}$$

$$\| (\mathit{everybody\ except\ who})(\mathrm{B}) \| = \mathrm{r} \Leftrightarrow (\mathrm{PERSON} - \mathrm{B})_{\mathrm{uk}} = \varnothing \tag{7}$$

Note that the right-hand side of the above all have the form $\mathrm{S}_{\mathrm{uk}} = \varnothing$ for an appropriate set S.

The semantics of IQs is more complicated than other GQs in that one does not only need to study the resolvedness conditions but also the resolved answers of IQs. For a typical WH-interrogative, the resolved answer may take two forms. The short form appears as a noun phrase. This form is called the constituent answer (CA). The full form appears as a complete sentence. This form is called the sentential answer (SA).

For strongly exhaustive IQs, it is easy to specify the semantic denotations of their CAs, as the form they take is closely related to their resolvedness conditions. For example, provided that $\| \mathit{which}(\mathrm{BOY}, \mathrm{SING}) \| = \mathrm{r}$, we have $(\mathrm{BOY} \cap \mathrm{SING})_{\mathrm{uk}} = \varnothing$, and the semantic denotation of the CA to "Which boy sang?" is then $(\mathrm{BOY} \cap \mathrm{SING})_{\mathrm{t}}$, i.e. all those entities who are known to be boys who sang. We can generalize the above: let q be a strongly exhaustive question whose resolvedness condition has the form $\mathrm{S}_{\mathrm{uk}} = \varnothing$, then the semantic denotation of the CA to 'q' is S_{t}.

Moreover, since SA is just the result of writing a CA in the form of a complete sentence, we can express the semantic denotation of an SA by making use of this relation as follows: let q be a strongly exhaustive question whose resolvedness condition has

5 Due to limited space, only the resolvedness conditions of a handful of IQs are given in this paper. The resolvedness conditions of other IQs may be derived in a similar fashion, although one needs to define some additional notions or domains for some IQs, such as a "possession" predicate for *whose,* a spatial domain for *where,* etc.

the form $S_{uk} = \varnothing$, then the semantic denotation of the SA to ‘q’ is the proposition $S = S_t$. Note that this proposition can often be re-expressed in the standard form as appears in the GQT literature by using the truth conditions of GQs. For illustration, suppose in a universe all entities who are known to be boys who sang are John and Bill, i.e. $(BOY \cap SING)_t = \{j, b\}$, and suppose $(BOY \cap SING)_{uk} = \varnothing$. Then the semantic denotation of the SA to “Which boy sang?” is the proposition BOY ∩ SING = {j, b}, which can be re-expressed as (*no ... except {j, b}*)(BOY, SING)[6]. This expression corresponds to the natural language sentence “No boy except John and Bill sang”.

An advantage of modeling WH-words as quantifiers is that we can derive the resolvedness conditions of WH-interrogatives involving predicates with an arity of 2 or higher by applying certain established operations in GQT. Under GQT, a sentence containing a higher-arity predicate can be viewed as containing a polyadic quantifier. There is an important subtype of polyadic quantifiers, called iterated quantifiers, whose truth conditions can be derived by using an operation called “iteration”. For example, consider the sentence “Every boy loves every girl” which contains a binary predicate “loves”. The truth condition of this sentence can be expressed as:

$$\| \textit{every}(\text{BOY}, \{x: \textit{every}(\text{GIRL}, \{y: \text{LOVE}(x, y)\})\}) \| = t \Leftrightarrow \text{BOY} \subseteq \{x: \text{GIRL} \subseteq \{y: \text{LOVE}(x, y)\}\} \quad (8)$$

The above formula says that “Every boy loves every girl” is true iff every boy x is such that for every girl y, x loves y.

A WH-interrogative containing a higher-arity predicate can be treated in a similar fashion. For illustration, consider the interrogative “Which girl does every boy love?”. According to the literature, this interrogative has at least 2 different readings: an individual reading and a pair-list reading. In this paper, I will only consider the individual reading, which can be paraphrased as “Which girl is such that every boy loves her?”[7]. Using iteration, one can easily derive the resolvedness condition of this reading as:

$$\| \textit{which}(\text{GIRL}, \{y: \textit{every}(\text{BOY}, \{x: \text{LOVE}(x, y)\})\}) \| = r \Leftrightarrow (\text{GIRL} \cap \{y: \text{BOY} \subseteq \{x: \text{LOVE}(x, y)\}\})_{uk} = \varnothing \quad (9)$$

The above formula says that “Which girl does every boy love?” is resolved iff there is no entity y such that it is not known whether y is a girl and is loved by every boy.

3.2 Non-Exhaustive IQs

Apart from “strongly exhaustive” interrogatives requesting complete information concerning a subject matter, there are also “non-exhaustive” interrogatives which request only partial information. [3] listed certain markers in natural languages and

---

6 According to the standard GQT literature, the truth condition of the GQ “(*no ... except C*)(A, B)” where C is a non-empty set of individuals manifested as (conjoined) proper names is $\| (\textit{no ... except } C)(A, B) \| = t \Leftrightarrow A \cap B = C$.

7 To handle the pair-list reading properly, we need more notions which are definable under the semantic model developed in this paper.

pointed out that interrogatives with these markers have inherent exhaustivity. For instance, “for example” is a marker of non-exhaustivity as exemplified by the interrogative “Which boy sang, for example?”. In this paper, WH-phrase “which … for example” will be expressed as a non-exhaustive IQ (*at least which*). A non-exhaustive WH-interrogative such as “Which boy sang, for example?” is resolved in two mutually exclusive situations: (1) at least one member of U is known to belong to BOY $\cap$ SING; (2) all members of U are known not to belong to BOY $\cap$ SING. Thus, the resolvedness condition can be written as $\|(\textit{at least which})(\text{BOY, SING})\| = \text{r} \Leftrightarrow (\text{BOY} \cap \text{SING})_t \neq \varnothing \vee (\text{BOY} \cap \text{SING})_f = \text{U}$. Note that situations (1) and (2) are represented by the two disjuncts on the right-hand side of this resolvedness condition. Generalizing the above discussion, the resolvedness conditions of two non-exhaustive IQs are given below:

$$\|(\textit{at least which})(\text{A, B})\| = \text{r} \Leftrightarrow (\text{A} \cap \text{B})_t \neq \varnothing \vee (\text{A} \cap \text{B})_f = \text{U} \tag{10}$$

$$\|(\textit{at least who})(\text{B})\| = \text{r} \Leftrightarrow (\text{PERSON} \cap \text{B})_t \neq \varnothing \vee (\text{PERSON} \cap \text{B})_f = \text{U} \tag{11}$$

Next I derive the semantic denotation of the CA to the non-exhaustive interrogative “Which boy sang, for example?”. Since the CA to a non-exhaustive interrogative is not unique, I will provide the set of the semantic denotations of all possible CAs, called the CA set, as follows:

$$\text{CA set} = \begin{cases} \{\text{X: X} \subseteq (\text{BOY} \cap \text{SING})_t \wedge \text{X} \neq \varnothing\}, & \text{if } (\text{BOY} \cap \text{SING})_t \neq \varnothing \\ \{\varnothing\}, & \text{if } (\text{BOY} \cap \text{SING})_f = \text{U} \end{cases} \tag{12}$$

The above piecewise-defined function provides the CA set under two mutually exclusive situations. If $(\text{BOY} \cap \text{SING})_f = \text{U}$, no boy sang and so the unique CA should be “none of them”, represented by a set consisting of $\varnothing$ as the unique member. If $(\text{BOY} \cap \text{SING})_t \neq \varnothing$, then every non-empty subset of $(\text{BOY} \cap \text{SING})_t$, i.e. any set X satisfying $\text{X} \subseteq (\text{BOY} \cap \text{SING})_t \wedge \text{X} \neq \varnothing$, is the semantic denotation of an acceptable CA. So all these Xs are collected into a set, and the CA can be represented by any member of this set. For illustration, suppose $(\text{BOY} \cap \text{SING})_t = \{\text{j, b}\}$, then the CA set is $\{\{\text{j}\}, \{\text{b}\}, \{\text{j, b}\}\}$, i.e. any one of “John”, “Bill” and “John and Bill” is an acceptable CA to the non-exhaustive interrogative “Which boy sang, for example?”.

Similar to CA, the SA to “Which boy sang, for example?” is also not unique and may be represented by a set of propositions, called the SA set, as shown below:

$$\text{SA set} = \begin{cases} \{\text{X} \subseteq \text{BOY} \cap \text{SING: X} \subseteq (\text{BOY} \cap \text{SING})_t \wedge \text{X} \neq \varnothing\}, & \text{if } (\text{BOY} \cap \text{SING})_t \neq \varnothing \\ \{\text{BOY} \cap \text{SING} = \varnothing\}, & \text{if } (\text{BOY} \cap \text{SING})_f = \text{U} \end{cases} \tag{13}$$

For illustration, suppose $(\text{BOY} \cap \text{SING})_t = \{\text{j, b}\}$, then the SA set is $\{\{\text{j}\} \subseteq \text{BOY} \cap \text{SING}, \{\text{b}\} \subseteq \text{BOY} \cap \text{SING}, \{\text{j, b}\} \subseteq \text{BOY} \cap \text{SING}\}$, i.e. any one of the sentences “John sang”, “Bill sang” and “John and Bill sang” is an acceptable SA to the non-exhaustive interrogative “Which boy sang, for example?”.

It is not hard to generalize (12) and (13) to a general non-exhaustive question q whose resolvedness condition has the form $S_t \neq \emptyset \vee S_f = U$ for an appropriate set S. All we need to do is replace the set BOY ∩ SING in (12) and (13) by S.

3.3 Polar Interrogatives

In this subsection I will discuss polar interrogatives. I propose that a polar interrogative be represented by *whether*(p) where *whether* is a Boolean operator asking for the truth value of p, where ‘p’ is the declarative associated with the polar interrogative. In this respect, *whether* is similar to the unary Boolean operator “¬”. While the latter may be manifested as “It is not the case that”, the former may be manifested as “Is it the case that”. For example, since the declarative associated with the polar interrogative “Does John love Mary?” is “John loves Mary”, the formal representation of this polar interrogative is *whether*(LOVE(j, m)).

Since a polar interrogative is resolved iff its associated declarative is known to be true or false, we can easily write down the resolvedness conditions for polar interrogatives:

$$\|\mathit{whether}(p)\| = r \Leftrightarrow \|p\| \neq uk \qquad (14)$$

I next determine the semantic denotations of the CA and SA to a polar interrogative. For a polar question *whether*(p), the semantic denotation of its CA can be easily written down as $\|p\|$. In English, $\|p\|$ can be represented by particular words, such as “yes” (corresponding to $\|p\| = t$) and “no” (corresponding to $\|p\| = f$). As for the semantic denotation of the SA to a polar interrogative, it can be expressed as

$$\text{Semantic denotation of SA} = \begin{cases} p, & \text{if } \|p\| = t \\ \neg p, & \text{if } \|p\| = f \end{cases} \qquad (15)$$

4 Interrogative Inferences

To study interrogative inferences, we need to define entailment and equivalence relations involving questions. Under the present framework, it is straightforward to define these notions. First, we define the notion of entailments: let $S = \{s_1, \dots s_n\}$ be a set of questions / propositions (called the premises) and q a question (called the consequence), then S entails q (denoted $S \Rightarrow q$) iff in every model, if $\|s_1\| \in \{t, r\}$ and … $\|s_n\| \in \{t, r\}$, then $\|q\| = r$.

Next we define the notion of equivalence: let q_1 and q_2 be questions, then q_1 is equivalent to q_2 (denoted $q_1 \Leftrightarrow q_2$) iff in every model, $\|q_1\| = r$ if and only if $\|q_2\| = r$.

4.1 Interrogative Entailments

Based on the resolvedness conditions of IQs and the above definitions, we can derive valid inferential patterns of IQs. We first consider some basic entailments:

$$\textit{which}(A, B) \Rightarrow (\textit{at least which})(A, B) \tag{16}$$

$$\textit{which}(A, B) \Rightarrow \textit{whether}(\textit{some}(A, B)) \tag{17}$$

These two entailments are in accord with our intuition. For example, if we know the answer to the strongly exhaustive interrogative “Which boy sang?”, we automatically know an answer to the non-exhaustive interrogative “Which boy sang, for example?” as well as the answer to the polar interrogative “Did any boy sing?”.

To prove (16) and (17), we first assume that $\|\textit{which}(A, B)\| = t$. By (4), this is true iff $(A \cap B)_{uk} = \emptyset$, i.e. for all $x \in U$, $\|x \in A \cap B\|$ is either equal to t or f. From this we can deduce that either there is an x such that $\|x \in A \cap B\| = t$, or for all x, $\|x \in A \cap B\| = f$, which is equivalent to the following two propositions:

$$(A \cap B)_t \neq \emptyset \vee (A \cap B)_f = U \tag{18}$$

$$\|A \cap B \neq \emptyset\| = t \vee \|A \cap B \neq \emptyset\| = f \tag{19}$$

From (18) we can then deduce $\|(\textit{at least which})(A, B)\| = r$ by (10) and thus complete the proof of (16). From (19) we can then deduce $\|A \cap B \neq \emptyset\| \neq uk$. By (14), we have $\|\textit{whether}(\textit{some}(A, B))\| = r$ and thus complete the proof of (17) since $A \cap B \neq \emptyset$ is the truth condition of *some*(A, B), according to the GQT literature.

Apart from inferences with only one premise, we may also consider interrogative inferences with more than one premise, such as the following:

$$\{\textit{which}(C, B), \textit{which}(C, A), A \subseteq C\} \Rightarrow \textit{which}(A, B) \tag{20}$$

Note that (20) is a generalization of a result in [5]. An instance of this inference schema is that the two questions “Which child does Mary teach?” and “Which child is a boy?” collectively entail the question “Which boy does Mary teach?” (on the understanding that boys are children).

To prove (20), we first write down the resolvedness conditions of the first two premises:

$$(C \cap B)_{uk} = \emptyset, (C \cap A)_{uk} = \emptyset \tag{21}$$

We then observe that $(C \cap B) \cap (C \cap A) = A \cap B$, given the third premise $A \subseteq C$. Next we need to apply the following result:

$$\text{For any sets A and B, } (A \cap B)_{uk} \subseteq A_{uk} \cup B_{uk}. \tag{22}$$

(22) can be proved as follows: let $x \in (A \cap B)_{uk}$, then $\|x \in A \cap B\| = uk$. This implies that $\|x \in A\| = uk$ or $\|x \in B\| = uk$. But this is equivalent to $x \in A_{uk}$ or $x \in B_{uk}$, i..e. $x \in A_{uk} \cup B_{uk}$.

Combining the above results, we have $(A \cap B)_{uk} \subseteq (C \cap B)_{uk} \cup (C \cap A)_{uk}$. By (21) we have $(A \cap B)_{uk} \subseteq \varnothing \cup \varnothing$, i.e. $(A \cap B)_{uk} = \varnothing$. The consequence of (20) thus obtains.

Monotonicity inferences constitute a special subtype of entailments. Monotonicity is concerned with truth preservation of a quantified statement when the arguments of the statement are replaced by their supersets / subsets. Here are the definitions of increasing and decreasing monotonicities: let $Q(X_1, \ldots X_n)$ be a GQ with n arguments, then Q is increasing in the i^{th} argument ($1 \le i \le n$) iff for all $X_1, \ldots X_i, X_i', \ldots X_n$ such that $X_i \subseteq X_i'$, $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow Q(X_1, \ldots X_i', \ldots X_n)$. Q is decreasing in the i^{th} argument iff for all $X_1, \ldots X_i, X_i', \ldots X_n$ such that $X_i \supseteq X_i'$, $Q(X_1, \ldots X_i, \ldots X_n) \Rightarrow Q(X_1, \ldots X_i', \ldots X_n)$. Q is called monotonic in the i^{th} argument iff it is either increasing or decreasing in the i^{th} argument. Otherwise, it is called non-monotonic in the i^{th} argument.

By treating WH-words as quantifiers, we may also talk about the monotonicities of WH-words. But the basic results turn out to be negative. First, according to the definition of entailments, “Which boy sang?” does not entail “Which boy sang Auld Lang Syne?”. The point is that even if you know exactly who sang, you may still not know exactly who sang Auld Lang Syne, because the latter interrogative requires more information than the former. In fact, we can show that[8]

Proposition 1 All strongly exhaustive IQs studied in this paper are non-monotonic in all of their arguments.

Here I will only prove *which* is not decreasing. The remaining part of the proof and the proofs for other strongly exhaustive IQs are similar. I construct a counterexample model. Let $U = A_t = \{a, b, c\}$, $A_f = A_{uk} = \varnothing$, $B_t = \{b, c\}$, $B_f = \{a\}$, $B'_t = \{b\}$, $B'_f = \{a\}$, $B'_{uk} = \{c\}$. It is obvious that this model satisfies $B \supseteq B'$ and $(A \cap B)_{uk} = \varnothing$. Thus, according to (4), $\| which(A, B) \| = r$. But since $(A \cap B')_{uk} \neq \varnothing$, we have $\| which(A, B') \| = ur$. The above fact shows that *which* is not decreasing.

I next consider the non-exhaustive IQs. According to (10) and (11), the resolvedness condition of each of these IQs is composed of two disjuncts. Due to this complexity, it turns out that all these IQs are in general non-monotonic in all of their arguments. For example, from the fact that all boys are children, we cannot deduce the following entailment:

$$(\textit{at least which})(\text{BOY, SING}) \Rightarrow (\textit{at least which})(\text{CHILD, SING}) \quad (23)$$

because it may be the case that the children in question consist of boys and girls, and it is known that no boy sang, while it is not known whether there was any girl who sang. In this case, the premise is resolved, but the consequence is not. The invalidity of (23) is mainly due to the fact that the resolvedness condition of the premise of (23)

[8] As a matter of fact, [7] contended that all IQs are decreasing. But his conclusion is based on his special definitions for monotonicities of IQs, which look very different from the usual definitions for monotonicities as used in the GQT literature. I thus do not adopt his definitions and obtain a different conclusion.

is composed of two disjuncts: “either at least one boy is known to have sung, or it is known that no boy sang”. If we now discard the second disjunct, then the resulting premise entails that at least one child is known to have sung. Thus in this case, (23) is valid.

The above discussion shows that the non-exhaustive IQs are in general non-monotonic, but may become increasing in certain specific cases. In fact, we have:

Proposition 2 Within the domain $\{<A, B>: (A \cap B)_f \neq U\}$, (*at least which*) is increasing in both of its arguments, whereas within the domain $\{B: (PERSON \cap B)_f \neq U\}$, (*at least who*) is increasing in its unique argument.

Here I will only prove that within the domain $\{<A, B>: (A \cap B)_f \neq U\}$, (*at least which*) is increasing in A. The remaining part of the proof and the proof for (*at least who*) is similar. Suppose $\|$(*at least which*)(A, B)$\|$ = r and $A \subseteq A'$. According to (10), within the given domain, $\|$(*at least which*)(A, B)$\|$ = r iff $(A \cap B)_t \neq \varnothing$. So we must have $(A' \cap B)_t \neq \varnothing$. This implies that $\|$(*at least which*)(A’, B)$\|$ = r, thus showing that (*at least which*) is increasing in A.

4.2 Interrogative Equivalences

Based on the definition of equivalences and the resolvedness conditions of IQs, we can easily derive (and prove) simple equivalences between IQs such as the following:

$$\textit{who}(B) \Leftrightarrow \textit{which}(PERSON, B) \tag{24}$$

$$\textit{which}(A, \neg B) \Leftrightarrow (\textit{all except which})(A, B) \tag{25}$$

These two equivalences are in accord with our intuition that “Who sang?” has the same meaning as “Which person sang?”[9], whereas “Which boy did not sing?” has the same meaning as “All except which boy sang?”

The equivalence in (25) involves “inner negation”, i.e. negation on an argument of the IQ. We now consider the notion of “outer negation”, i.e. negation of a question, and see if we can derive any equivalence[10]. Our first problem is whether we can make a proper definition for the negation of a question which should conform to the requirement of the negation operator “¬” in the definition of bilattices set out in Section 2, i.e. “¬” should preserve the order in the interrogative lattice. One way to achieve this is to define “¬” such that for any question q,

$$\|\neg q\| = r \Leftrightarrow \|q\| = r \tag{26}$$

For example, [10, 11] stipulated that “¬” has null effect on questions, which is equivalent to defining $\neg q = q$. But this definition runs counter to our intuition about nega-

9 This is true only if we ignore the context-dependent effect of *which*.

10 The formal definitions of inner negation and outer negation can be found in [12].

tion. Therefore, I will try to provide alternative definitions for negated questions which satisfy (26). I will consider WH-questions and polar questions in turn.

First consider a strongly exhaustive WH-question q asking for S, where S is a certain set. Its outer negation ¬q can be defined as another strongly exhaustive WH-question asking for ¬S. Now the resolvedness condition of q is $S_{uk} = \varnothing$. When $S_{uk} = \varnothing$, we have $S = S_t$ and $\neg S = S_f$. Thus, S_t and S_f are the semantic denotations of the resolved CAs to ‘q’ and ‘¬q’, respectively. In contrast, when $S_{uk} \neq \varnothing$, we cannot determine S and ¬S because we do not know whether the elements in S_{uk} belong to S or ¬S. Thus, $S_{uk} = \varnothing$ is both a resolvedness condition of q and ¬q, and so (26) is satisfied.

However, under the above definition, the outer negation of a strongly exhaustive WH-question often results in an unnatural question. For example, the outer negation of “Which boy sang” is something like the following:

$$\text{Which individual was not a boy who sang?} \qquad (27)$$

Note that the above is a rather strange way to form an interrogative. It is completely different from the natural interrogative “Which boy did not sing?”. While the latter asks for $BOY \cap \neg SING$, the former asks for $\neg(BOY \cap SING)$. Since the outer negation of a strongly exhaustive WH-question is unnatural, no sensible equivalence can be derived for this type of questions[11].

Next consider a polar question *whether*(p). Its outer negation can be defined as the polar question *whether*(¬p)[12]. Now the resolvedness condition of *whether*(p) is $\|p\| \neq uk$, which is equivalent to $\|p\| \in \{t, f\}$. This last statement is true iff $\|\neg p\| \in \{t, f\}$, which is equivalent to $\|\neg p\| \neq uk$. Thus we have the following equivalence:

$$\textit{whether}(p) \Leftrightarrow \textit{whether}(\neg p) \qquad (28)$$

and (26) is satisfied.

Under the above definition, the outer negation of a polar interrogative is its negative counterpart. For example, the outer negation of “Does John love Mary?” is “Doesn’t John love Mary?”, provided that this negative polar interrogative is not read as a rhetorical question.

According to [9], we can derive logical equivalences by combining the inner negation and outer negation of different GQs. For example, [9] proposed the following valid inference schema:

$$Q_1(A, \{x: Q_2(B, \{y: P(x, y)\})\}) \Leftrightarrow (Q_1\neg)(A, \{x: \neg Q_2(B, \{y: P(x, y)\})\}) \qquad (29)$$

---

[11] In principle, it is also possible to define outer negation for non-exhaustive WH-questions. But the result is even more bizarre. Given limited space, I will not discuss this issue in this paper.

[12] Although “¬” appears inside the argument position of *whether*, *whether*(¬p) should be seen as the outer negation of *whether*(p), because *whether*(¬p) satisfies the definition of outer negation.

where $Q_1\neg$ represents the inner negation of Q_1, whereas $\neg Q_2$ represents the outer negation of Q_2. If we now substitute a suitable IQ for Q_1 and an ordinary GQ for Q_2 in (29), we will obtain an equivalence relation involving both IQs and ordinary GQs, such as the following:

which(A, {x: *some*(B, {y: P(x, y)})}) ⇔ (*all except which*)(A, {x: *no*(B, {y: P(x, y)})}) (30)

In the above, I have made use of the fact that (*all except which*) is the inner negation of *which* whereas *no* is the outer negation of *some*. The above schema may be exemplified by a concrete example:

Which boy has got some prize? ⇔ All except which boy has got no prize? (31)

Note that the above is a sensible equivalence.

4.3 Answerhood

According to [10, 11], there is a special entailment relation between an interrogative and the SA to that interrogative. Let p be a proposition and q a question. Then we have the following[13]:

Proposition 3 If ‘p’ is an SA to ‘q’, then $p \Rightarrow q$.

To prove this, I have to consider several cases. When q is a polar question in the form *whether*(s), then p is either s or ¬s. Let $\|p\| = t$, which entails $\|s\| = t$ or $\|s\| = f$. In either case, $\|s\| \neq uk$. Then by (14), we have $\|q\| = r$. When q is a strongly exhaustive WH-question whose resolvedness condition has the form $S_{uk} = \varnothing$, p has the form $S = S_t$. Now let $\|p\| = t$, i.e. $\|S = S_t\| = t$. According to the definition of S_t, we have for all x, $\|x \in S_t\|$ is either equal to t or f. This is equivalent to $(S_t)_{uk} = \varnothing$. But since $S = S_t$, we have $S_{uk} = \varnothing$. Thus the resolvedness condition of q is satisfied, and so $\|q\| = r$. When q is a non-exhaustive WH-question whose resolvedness condition has the form $S_t \neq \varnothing \vee S_f = U$, p has the form $X \subseteq S$ (where $X \subseteq S_t \wedge X \neq \varnothing$) or $S = \varnothing$. Now let $\|p\| = t$, i.e. either $\|X \subseteq S\| = t$ or $\|S = \varnothing\| = t$. In the former case, we have $S_t \neq \varnothing$. In the latter case, we have $S_f = U$. In either case, the resolvedness condition of q is satisfied, and so we have $\|q\| = r$.

Proposition 3 shows that $p \Rightarrow q$ is a necessary condition for ‘p’ is an SA to ‘q’. In other words, we can show that ‘p’ is not an SA to ‘q’ by showing that p #⇒ q. For instance, we can show that “John sang” is not a resolved SA to “Which boy sang?” by proving that “John sang” (assuming that “John” is a boy) does not entail “Which boy sang?”. To prove this, we may construct a counterexample model. Let $U = BOY_t = \{j, b\}$, $BOY_f = BOY_{uk} = \varnothing$, $SING_t = \{j\}$, $SING_f = \varnothing$, $SING_{uk} = \{b\}$. With respect to this model, on the one hand, we find that $\|j \in SING\| = t$, i.e. “John sang” is true. On the other hand, we also find that $\|b \in BOY \cap SING\| = uk$, which shows that (BOY ∩

[13] The central idea of the following proposition is from [11]. But the proof is my own and is presented in terms of the definitions and results in this paper.

SING)$_{uk}$ $\neq \varnothing$. According to (4), we have $\|$*which*(BOY, SING)$\|$ $\neq$ r. Thus, "John sang" does not entail "Which boy sang?", and so the former is not a resolved SA to the latter.

5 Conclusion

In Section 1, I have mentioned 3 adequacy criteria which an adequate framework for interrogatives should satisfy. It is now time to see if the semantic model developed in this paper satisfies these criteria.

It is clear that the semantic model is materially adequate, as the notions of answerhood, entailments and equivalences are all definable under the model. Not only have I provided the resolvedness conditions for various types of interrogatives, but I have also provided explicit expressions for the semantic denotations of the CAs and SAs corresponding to these interrogatives.

The model is also formally adequate, as the semantic notions are interpretable as set-theoretic relations / operations. This point is particularly obvious as the resolvedness conditions and the semantic denotations of the CAs and SAs corresponding to various types of interrogatives are all expressed as set-theoretic relations or operations.

Finally, the model is also empirically adequate in that certain inferential relations that are intuitively correct are provable under the model. These include the interrogative entailments and equivalences recorded in Section 4 as well as Propositions 1 and 2 concerning the monotonicities of IQs.

References

1. Arieli, O., Avron, A.: Reasoning with Logical Bilattices. Journal of Logic, Language and Information. 5(1), 25-63 (1996)
2. Belnap, N.D. jr., Steel, T.B. jr.: The Logic of Questions and Answers. Yale University Press, New Haven (1976)
3. Beck, S., Rullmann, H.: A Flexible Approach to Exhaustivity in Questions. Natural Language Semantics. 7, 249-298 (1999)
4. Ginzburg, J.: Resolving Questions I and II. Linguistics and Philosophy. 18, 459-527, 567-609 (1995)
5. Groenendijk, J., Stokhof, M.: Type-shifting rules and the semantics of interrogatives. In Portner, P., Partee, B.H. (eds.) Formal Semantics: the Essential Readings, pp. 421-456. Blackwell, Oxford (1989)
6. Groenendijk, J. and Stokhof, M.: Questions. In van Benthem, J., ter Meulen, A. (eds.) Handbook of Logic and Language (2nd edition), pp. 1059 – 1132. Elsevier Science, Amsterdam, (2011)
7. Gutierrez-Rexach, J.: Questions and Generalized Quantifiers. In Szabolcsi, A. (ed.) Ways of Scope Taking, pp. 409-452. Kluwer Academic Publishers, Dordrecht (1997)
8. Gutierrez-Rexach, J.: Interrogatives and Polyadic Quantification. In Scott, N. (ed.) Proceedings of the International Conference on Questions, pp. 1-14. University of Liverpool (1999)

9. Keenan, E.L.: Excursions in Natural Logic. In Casadio, C. *et al* (eds.) Language and Grammar: Studies in Mathematical Linguistics and Natural Language, pp. 31-52. CSLI, Stanford (2003)
10. Nelken, R., Francez, N.: The Algebraic Semantics of Interrogative NPs. Grammars. 3, 259-273 (2000)
11. Nelken, R., Francez, N.: Bilattices and the Semantics of Natural Language Questions. Linguistics and Philosophy. 25, 37-64 (2002)
12. Peters, S., Westerståhl, D. Quantifiers in Language and Logic. Clarendon Press, Oxford (2006)