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Scope Ambiguity of Polar / Alternative Questions

Ka-fat CHOW

In Chow (2019), I developed a framework that combines the notions and notations of Inquisitive Semantics (IS) (as in Ciardelli *et al* (2019), among others) and Generalized Quantifier Theory (GQT) (as in Peters & Westerståhl (2006), among many others). On the one hand, this framework inherits IS's practice of treating propositions as having type $(st)t$ (hereinafter abbreviated as T) and n-ary predicates as having type e^nT. It also inherits the various operations of IS, including the join and meet operations, the absolute pseudo-complement operator (represented as $\sim$ in this paper), which is the negation operator under IS, and two projection operators: the ! operator, which turns a proposition to an assertion, and the ? operator, which turns a proposition to a question. The definitions of the last three operators are given below:

$$\sim p = \text{Power}(W - \bigcup p) \qquad (1)$$

$$!p = \text{Power}(\bigcup p) \qquad (2)$$

$$?p = p \cup \sim p \qquad (3)$$

where Power represents the power set operation and W represents the set of all possible worlds.

On the other hand, this framework also tries to incorporate the classical treatment of GQT for generalized quantifiers (GQs) under the IS framework. It proposes an additional type for n-ary predicates, namely $s(e^nt)$. Thus, for each predicate X of the standard type e^nT, there is a corresponding predicate X^* of the additional type $s(e^nt)$. The following formulae are used for switching between X and X^* (where w and x are variables of types s and e^n, respectively):

$$X^* = \lambda w[\{x : \{w\} \in X(x)\}] \qquad (4)$$

$$X = \lambda x[\text{Power}(\{w : x \in X^*(w)\})] \qquad (5)$$

Using the additional type, we can then denote GQs in a way that resembles the classical denotations of GQs under GQT. Let Q' be a monadic GQ under the classical GQT with the denotation $Q' = \lambda X'_1 \ldots \lambda X'_n[C(X'_1, \ldots, X'_n)]$ where $X'_1, \ldots, X'_n$ are variables of type et and C is the truth condition associated with this GQ. Then there is a corresponding monadic GQ Q under IS with the following denotation (where $X_1, \ldots, X_n$ are variables of type eT):

$$Q = \lambda X_1 \ldots \lambda X_n[\mathrm{Power}(\{w : C(X_1^*(w), \ldots, X_n^*(w))\})] \qquad (6)$$

Based on the aforesaid framework, in this paper I first propose the proper treatments of polar/alternative questions containing GQs (this paper mainly discusses alternative questions with two noun phrases as choices, i.e. the terms connected by "or" in an alternative question). Regarding polar questions, I propose that the denotations of such kind of questions should have the following general form:

$$?!p \qquad (7)$$

where $!p$ represents the declarative sentence associated with the polar question (the operator ! is used to suppress any inquisitiveness which p may have).

Regarding alternative questions, according to Roelofsen & van Gool (2010), Pruitt & Roelofsen (2011), Biezma & Rawlins (2012) and Steiner-Mayr (accepted 2024), the choices in such kind of questions are subject to a "true alternative requirement" according to which the addressee must choose one of the choices in the question as answer, as well as a "mutual exclusiveness requirement" according to which the addressee can only choose one of the choices as answer.

To meet the aforesaid two requirements, I propose the following treatment of alternative questions which contains two main points. First, I propose a "higher order modifier" (borrowing a term from Zuber (1997)) called PREC, short for "precisification", which acts on the choices of an alternative question and turns them to their "precisified" version, with the following denotation (where Q is a variable of type $\langle 1 \rangle$ GQs and X is a variable of type eT):

$$\textsc{prec} = \lambda Q \lambda X[\mathrm{Power}(\{w : X^*(w) \in \mathrm{Wit}(Q)(w)\})] \qquad (8)$$

In the expression above, Q is a type $\langle 1 \rangle$ GQ, $\mathrm{Wit}(Q)$ represents a function mapping a world to the set of witness sets of Q in that world. The formal definition of "witness sets" can be found in Chow (2024). Roughly speaking, a witness set of Q in a world w is a set that can serve as a representative of Q in w. For example, in a model that contains the predicate **mi** (representing "musical instrument"), the witness sets of $\textsc{a}(\mathbf{mi})$ in w are all those sets that precisely contain at least a member of $\mathbf{mi}^*(w)$. Since the members of $\mathbf{mi}^*(w)$ may differ as w varies, $\mathrm{Wit}(\textsc{a}(\mathbf{mi}))$ is a function dependent on possible worlds.

Second, I propose that the aforesaid two requirements give rise to a presupposition whose purpose is to restrict the set W of all possible worlds to those that satisfy these requirements and I treat this presupposition as an additional expression appended after the core denotation of the alternative question.

In the light of the above discussion, I now write down the canonical form of the denotation of an alternative question as follows:

$$d = \textsc{or}(\textsc{prec}(Q_1), \textsc{prec}(Q_2))(X); \;\; W = \bigcup d \qquad (9)$$

where d represents the core denotation of the alternative question, Q_1 and Q_2 are type $\langle 1 \rangle$ GQs representing the two choices, X is a unary predicate, and OR has the following recursive definition (adapted from Winter (2001) and Ciardelli *et al* (2019)):

$$\text{OR} = \begin{cases} \lambda(p_1, p_2)[p_1 \cup p_2], & \text{if } p_1,\ p_2 \text{ have type } T \\ \lambda(X_1, X_2)\lambda Y[\text{OR}(X_1(Y), X_2(Y))], & \text{if } X_1,\ X_2 \text{ have type } \tau_1\tau_2 \text{ and } Y \text{ has type } \tau_1 \end{cases} \quad (10)$$

In the first expression in (9), which is the core denotation, the operator OR occupies the first position without falling under the scope of any GQ because it is precisely this operator that gives rise to the inquisitiveness of the question. If it falls under the scope of a GQ, its inquisitiveness will be suppressed by the GQ. The second expression in (9), which is the presupposition, states that W is the union of the members of d and so contains all those worlds that satisfy the aforesaid two requirements.

I then discuss the scope ambiguity of certain types of questions. As a matter of fact, Chow (2019) discussed the much-studied scope ambiguity of certain constituent questions such as "Which book did every girl read?" and proposed a proper treatment for its two readings, namely the "individual reading" and the "pair list reading". Instead of discussing such kind of scope ambiguity, in this paper I will discuss the less-studied scope ambiguity of certain polar/alternative questions.

To treat scope properly, I adopt the standard GQT practice of treating GQs as arity reducers (as in Peters & Westerståhl (2006)) as well as case extension operators with the following definitions adapted from Keenan (1987) (where nom and acc represent the nominative and accusative case extensions respectively, Q is a type $\langle 1 \rangle$ GQ, R is a variable of type e^2T, and x and y are variables of type e):

$$Q_{\text{nom}} = \lambda R \lambda y[Q(\lambda x[R(x, y)])] \quad (11)$$

$$Q_{\text{acc}} = \lambda R \lambda x[Q(\lambda y[R(x, y)])] \quad (12)$$

It has been shown in Chow (2019) that by using (4), (5), (6), (11) and (12), one can derive the correct denotations of quantified statements with iterated GQs under the IS framework. In fact, the treatment can even be extended to quantified statements containing certain "higher order modifiers" or "generalized noun phrases" discussed in Zuber (1997), Zuber (2018), Zuber (2019), Chow (2024), etc.

In addition, I further propose the following "scope reversal cum case extension" version of Q (where rv is short for "reversal", nom/acc is a variable label which may be instantiated as either nom or acc, P is a variable of type $\langle 1 \rangle$ GQs and R is a variable of type e^2T):

$$Q_{\text{rv,nom/acc}} = \lambda P \lambda R[P(Q_{\text{nom/acc}}(R))] \quad (13)$$

By applying the operator Q above (with the labels suppressed) to a GQ P, we obtain $P(Q(...))$, hence reversing the scope relation between Q and P.

There are two sources of ambiguity in polar/alternative questions. The first source is the scope structures of certain GQs. According to the study of GQT and other related studies, a declarative sentence with certain GQs at the subject and object positions of the sentence such as "A kid climbed every tree" is ambiguous between an "object narrow scope (ONS) reading" under which the GQ at the object position takes a narrower scope than that at the subject position, and an "object wide scope (OWS) reading" under which the GQ at the object position takes a wider scope than that at the subject position. A polar question with the same GQ structure such as the following also exhibits the same scope ambiguity:

Did a kid climb every tree? (14)

The ONS and OWS readings of (14) can be represented by using the general form of polar questions given in (7) above and the operators (11), (12) and (13) introduced above as follows:

$$?!\textsc{a}(\mathbf{kid})(\textsc{every}(\mathbf{tree})_{\mathrm{acc}}(\mathbf{climbed})) \qquad (15)$$

$$?!\textsc{a}(\mathbf{kid})_{\mathrm{rv,nom}}(\textsc{every}(\mathbf{tree}))(\mathbf{climbed}) \qquad (16)$$

In (16) above, I use the operator $\textsc{a}(\mathbf{kid})_{\mathrm{rv,nom}}$ because under the OWS reading, the subject "a kid" in (14) takes a narrower scope than the object "every tree". By substituting (13) into (16), (16) will finally turn out to be an expression with $\textsc{every}(\mathbf{tree})$ taking a wider scope than $\textsc{a}(\mathbf{kid})$.

The second source is the inherent ambiguity of the word "or" such as the following:

Did John learn a musical instrument or a foreign language? (17)

The question above is ambiguous between a polar question (which expects the answer "yes" or "no" according as it is or it is not the case that John learned a musical instrument or a foreign language) and an alternative question (which expects the answer "a musical instrument" or "a foreign language"). As I have discussed the proper treatments of polar/alternative questions above, the polar question and alternative question readings of (17) can be represented as follows (where $\mathrm{I}_{\mathbf{j}}$ is the Montagovian individual representing "John"; moreover, the presuppositions of all alternative questions are omitted below as they can easily be determined from the core denotations of these questions):

$$?!\mathrm{I}_{\mathbf{j}}(\textsc{or}(\textsc{a}(\mathbf{mi}), \textsc{a}(\mathbf{fl}))_{\mathrm{acc}}(\mathbf{learned})) \qquad (18)$$

$$d = \textsc{or}(\textsc{prec}(\textsc{a}(\mathbf{mi})), \textsc{prec}(\textsc{a}(\mathbf{fl})))(\mathrm{I}_{\mathbf{j}\mathrm{nom}}(\mathbf{learned})) \qquad (19)$$

The aforesaid two sources of ambiguity can even be exhibited in a single question such as the following:

Must a musical instrument or a foreign language be taught to every student? (20)

Since this question contains “a” and “every” at the subject and object positions respectively, it is ambiguous between an ONS reading and an OWS reading. Moreover, since it contains “or”, it is also ambiguous between a polar question and an alternative question. Thus, this question is ambiguous with at least four readings.

The ONS polar question reading and OWS polar question reading of (20) can be represented in a way similar to (15) and (16) above (in what follows, I treat “must be taught to” as a whole unit and represent it by **mbtt**, ignoring its internal structure):

$$?!\textsc{or}(\textsc{a}(\mathbf{mi}), \textsc{a}(\mathbf{fl}))(\textsc{every}(\mathbf{student})_{\mathrm{acc}}(\mathbf{mbtt})) \qquad (21)$$

$$?!\textsc{or}(\textsc{a}(\mathbf{mi}), \textsc{a}(\mathbf{fl}))_{\mathrm{rv,nom}}(\textsc{every}(\mathbf{student}))(\mathbf{mbtt}) \qquad (22)$$

The ONS alternative question reading of (20) can be represented in a way similar to (19) above:

$$d = \textsc{or}(\textsc{prec}(\textsc{a}(\mathbf{mi})), \textsc{prec}(\textsc{a}(\mathbf{fl})))(\textsc{every}(\mathbf{student})_{\mathrm{acc}}(\mathbf{mbtt})) \qquad (23)$$

Note that in the expression above, the GQs $\textsc{a}(\mathbf{mi})$ and $\textsc{a}(\mathbf{fl})$ take a wider scope than $\textsc{every}(\mathbf{student})$, which agrees with the ONS reading of (20).

As for the OWS alternative question reading of (20), the situation is a bit more complicated. On the one hand, we must place the operator $\textsc{or}$ at the first position of the representation as dictated by the requirement that its inquisitiveness should not be suppressed by other operators. On the other hand, we also need to place $\textsc{every}(\mathbf{student})$ in front of $\textsc{a}(\mathbf{mi})$ and $\textsc{a}(\mathbf{fl})$ because under the OWS reading, “every student” takes a wider scope than “a musical instrument” and “a foreign language”.

To resolve the aforesaid paradox, we can make use of the “scope reversal cum case extension” version of $\textsc{prec}(\textsc{a}(\mathbf{mi}))$ and $\textsc{prec}(\textsc{a}(\mathbf{fl}))$ and represent the OWS alternative question reading of (20) in the following non-canonical form:

$$d = \textsc{or}(\textsc{prec}(\textsc{a}(\mathbf{mi}))_{\mathrm{rv,nom}}, \textsc{prec}(\textsc{a}(\mathbf{fl}))_{\mathrm{rv,nom}})(\textsc{every}(\mathbf{student}))(\mathbf{mbtt}) \qquad (24)$$

If we substitute (10), (11) and (13) into the expression above and then apply λ-reduction, we will obtain the following as an intermediate result:

$$\begin{aligned} & \textsc{every}(\mathbf{student})(\textsc{prec}(\textsc{a}(\mathbf{mi}))_{\mathrm{nom}}(\mathbf{mbtt})) \\ \cup \quad & \textsc{every}(\mathbf{student})(\textsc{prec}(\textsc{a}(\mathbf{fl}))_{\mathrm{nom}}(\mathbf{mbtt})) \end{aligned} \qquad (25)$$

The expression above has the form $p \cup q$, and so it is inquisitive. Moreover, in both p and q, the GQ $\textsc{every}(\mathbf{student})$ takes a wider scope than $\textsc{a}(\mathbf{mi})$ and $\textsc{a}(\mathbf{fl})$, and so this represents the OWS reading of (20). Thus, (24) is a correct representation of the OWS alternative question reading of (20).

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