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Internal Reading and Reciprocity

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Sentences with "each other" in English have been treated as the canonical reciprocal expressions. However, apart from "each other", some other sentences also carry a reciprocal meaning. A typical example is the following sentence:

John, Peter and Mary read the same books. (1)

The phrase with "the same" above is ambiguous with two readings: the deictic reading and the internal reading, which as argued in [3], are different readings without one being a special case of the other. Under the deictic reading, "same" means "same as something mentioned in the previous discourse or salient in the context", whereas under the internal reading, the sentence above is equivalent to the following (as pointed out by [7]):

John, Peter and Mary read the same books as each other. (2)

meaning that any two of the three persons read the same books (here I assume the strongest reciprocal meaning as discussed in [5] by default). What I am interested is the internal reading because it is this reading that carries a reciprocal meaning.

In addition to "the same", some other lexical items, hereinafter called "reciprocal items" and including "different", "opposite", "similar", "related", "separate", "adjacent", "complementary", "connected", "disjoint", etc, behave similarly in that a sentence containing one of these items has an internal reading carrying a reciprocal meaning. This is best illustrated by the following example which sounds weird:

*Particles 1, 2 and 3 have different charges. (internal reading) (3)

Under the intended (namely internal) reading, the sentence above means that the charges of the three particles are different from each other, i.e. particles 1 and 2 have different charges, particles 2 and 3 have different charges, and particles 3 and 1 have different charges. But this is impossible given that there are only two possible charges (positive vs negative).

Another evidence for the reciprocal meaning of these items is that many of them can be rendered in Chinese as words containing the morpheme "xiang" or "hu" meaning "mutual", e.g. "same" as "xiangtong", "different" as "xiangyi", "opposite" as "xiangfan", "similar" as "xiangsi", "related" as "xiangguan", "separate" as "xiangge", "adjacent" as "xianglin", "complementary" as "hubu", "connected" as "hutong", "disjoint" as "huchi", etc.

While the internal reading of "the same" has been studied by [3], [4], [7], [10], [13], etc from different perspectives using different theoretical frameworks, none of these studies based the semantics of "the same" on reciprocity. Neither did these studies consider other reciprocal items (except "different"). In this paper, I will provide a uniform formal treatment for the internal reading of the reciprocal items that reflects the reciprocal meaning.

For the formal framework of this paper, I will adopt the notation in [9], which uses small cap font and boldface font for the denotations of logical and non-logical terms, respectively. If f is a characteristic function, then f^* is the set of entities that f characterizes. Moreover, I will also borrow ideas from the Generalized Quantifier Theory (GQT) enriched by the notion of generalized noun phrases (GNPs) developed by [11], [12], [14], etc.

Under the classical GQT, a generalized quantifier (GQ) is a function with a number of predicates as arguments and a truth value as output. The arities of the arguments are used to name the type of a GQ. For example, a determiner, such as "every", is treated as a type $\langle 1, 1 \rangle$ GQ because it requires a nominal unary predicate and a verbal unary predicate as its arguments.

The notion of GNP is a generalization of GQ. Instead of outputting truth values, a GNP outputs a set (or characteristic function) of ordered n-tuples of individuals (represented by ": n") or a set (or characteristic function) of type $\langle 1 \rangle$ GQs (represented by ": $\langle 1 \rangle$"). For example, according to [12], "each other" should be treated as a GNP with a binary predicate as input and a set of type $\langle 1 \rangle$ GQs as output, and its type is denoted $\langle 2 : \langle 1 \rangle \rangle$.

Following [12], I will treat "each other" as a type $\langle 2 : \langle 1 \rangle \rangle$ GNP. As for the truth condition of this GNP, I will adopt a modified version of the truth condition of the FUL quantifier studied in [6] which is given as follows:

$$\textsc{each other} = \lambda R \lambda Q [\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq R^*]] \tag{4}$$

In the above, R and Q are variables of binary predicates and upward monotonic type $\langle 1 \rangle$ GQs, respectively. $\mathrm{Wit}(Q)$ represents the set of witness sets of Q, whose definition can be found in [8]. Id_X^2 is the set $\{(x, x) : x \in X\}$ and so the statement $X^2 - Id_X^2 \subseteq R^*$ means that every member x of X stands in the relation R with every other member of X except perhaps x itself. For instance, the following sentence

John, Peter and Mary hit each other. (5)

will be denoted by

$$\textsc{each other}(\mathbf{hit})(\textsc{and}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{6}$$

where $I_{\mathbf{j}}$, $I_{\mathbf{p}}$ and $I_{\mathbf{m}}$ are the type $\langle 1 \rangle$ quantifiers (also called "Montagovian individuals" in the GQT literature) corresponding to the individual terms $\mathbf{j}$

(short for "John"), **p** (short for "Peter") and **m** (short for "Mary"), respectively. For simplicity, here I use a polymorphic boolean operator AND which takes any number of type $\langle 1\rangle$ quantifiers as input and output the intersection of them. According to (4), the expression above is true if and only if there exists a plural witness set of $\text{AND}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$ (which, according to the definition of witness sets, consists of the three individuals John, Peter and Mary) such that every member of that witness set hits every other member of that set, which is exactly what (5) means.

I propose that the internal reading of sentences containing the reciprocal items should also be treated using (4). For example, in order to use (4) to denote (1), we have to determine its argument R, which should be a binary predicate denoting "read the same books as". First, we determine the denotation of "the same ... as". According to [1], the structure "the same ... as" in the sentence "The same boys sang as danced" can be seen as a type $\langle 1, 1^2\rangle$ GQ with the following truth condition:

$$\text{THE SAME AS} = \lambda A\lambda(B, C)[\text{THE SAME}(A^* \cap B^*, A^* \cap C^*)] \tag{7}$$

where THE SAME is a logical term with the following denotation:

$$\text{THE SAME} = \lambda(A, B)[A = B] \tag{8}$$

While the structure "the same ... as" in the phrase "read the same books as" cannot be seen as a GQ, it was analyzed as a "generalized comparative determiners" (a subtype of GNPs) in [11]. Modifying the notion of generalized comparative determiners a bit in this paper, I provide the denotation of "the same ... as" in "read the same books as" as follows:

$$(\text{THE SAME AS})_2 = \lambda X\lambda R\lambda(u, v)[\text{THE SAME}(X^*\cap\{w : R(u, w) = 1\}, X^*\cap\{w : R(v, w) = 1\})] \tag{9}$$

where the subscript "2" means that "the same ... as" appears in the 2nd argument position (i.e. object) of the binary predicate R. Note that the variable w representing the noun modified by "the same" does appear in the 2nd argument position of R in the formula above. $(\text{THE SAME AS})_2$ as defined above is a type $\langle 1, 2 : 2\rangle$ GNP, with a unary predicate and a binary predicate as inputs and a set of ordered pairs of individuals as output. Having defined the GNP above, we then apply it to the unary predicate **book** and the binary predicate **read** to obtain the denotation of "read the same books as" as follows:

$$(\text{THE SAME AS})_2(\mathbf{book})(\mathbf{read}) \tag{10}$$

Finally, we can determine the denotation of (1) as follows:

$$\text{EACH OTHER}((\text{THE SAME AS})_2(\mathbf{book})(\mathbf{read}))(\text{AND}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{11}$$

By using (4), (9) and (8), one can show that the expression above is true if and only if the following is true:

$$\{(\mathbf{j},\mathbf{p}),(\mathbf{j},\mathbf{m}),(\mathbf{p},\mathbf{j}),(\mathbf{p},\mathbf{m}),(\mathbf{m},\mathbf{j}),(\mathbf{m},\mathbf{p})\}$$
$$\subseteq \{(u,v):\mathbf{book}^*\cap\{w:\mathbf{read}(u,w)=1\}=\mathbf{book}^*\cap\{w:\mathbf{read}(v,w)=1\}\} \quad (12)$$

The expression above means that any two (different) individuals among John, Peter and Mary are such that the books that one read are the same as the books that the other read, which is exactly what (1) means.

The internal reading of sentences containing the other reciprocal items can also be treated analogously. More specifically, I propose that all these items (each with a suitable preposition) should be treated as generalized comparative determiners of type $\langle 1,2:2\rangle$. For example, "separate ... from" is denoted by the following generalized comparative determiner:

$$(\textsc{separate from})_2 = \lambda X\lambda R\lambda(u,v)[\mathbf{separate}(X^*\cap\{w:R(u,w)=1\},X^*\cap\{w:R(v,w)=1\})] \quad (13)$$

A justification for the treatment above is that (13) satisfies a modified version of the property called "argument invariance for unary determiners" (D1AI) studied in [11], which can be seen as a defining property of such kind of GNPs. The modified definition of D1AI is given as follows: a type $\langle 1,2:2\rangle$ GNP F satisfies D1AI iff for any unary predicate X, binary predicate R and individuals a, b, c, d, if $X^*\cap\{w:R(a,w)=1\}=X^*\cap\{w:R(b,w)=1\}$ and $X^*\cap\{w:R(c,w)=1\}=X^*\cap\{w:R(d,w)=1\}$, then $F(X)(R)(a,c)=F(X)(R)(b,d)$.

The only difference between (9) and (13) is that while THE SAME is a logical term with a denotation independent of models like (8), **separate** is a non-logical term whose denotation is dependent on models, i.e. $\mathbf{separate}(A,B)=1$ in a particular model if and only if A^* and B^* are singletons and the unique members in these two singletons are separate entities in that model. Note that "separate ... from" is structurally similar to "older ... than", which was discussed and considered as a generalized comparative determiner in [11].

Based on the above, the denotation of the following sentence

John, Peter and Mary live on separate floors. (14)

can be determined as follows:

$$\textsc{each other}((\textsc{separate from})_2(\mathbf{floor})(\mathbf{live\ on}))(\textsc{and}(I_{\mathbf{j}},I_{\mathbf{p}},I_{\mathbf{m}})) \quad (15)$$

Apart from $\textsc{and}(I_{\mathbf{j}},I_{\mathbf{p}},I_{\mathbf{m}})$, other upward monotonic type $\langle 1\rangle$ GQs can also be used as the Q argument of (4). For example, by replacing $\textsc{and}(I_{\mathbf{j}},I_{\mathbf{p}},I_{\mathbf{m}})$ with AT LEAST $3(\mathbf{person})$ in (11), we obtain the denotation of the internal reading of the sentence "At least three persons read the same books".

For downward monotonic or non-monotonic type $\langle 1\rangle$ GQs, the truth condition given in (4) is not adequate. To obtain the appropriate truth conditions in such cases, I propose to borrow ideas from [2] and modify (4) as follows:

$$\text{EACH OTHER} = \lambda R \lambda Q \;\; [(\exists X \in \mathrm{Wit}(Q)[|X| \geq 2 \wedge X^2 - Id_X^2 \subseteq R^*] \vee \mathrm{Top}(Q) \cap S = \emptyset) \wedge Q(S) = 1] \tag{16}$$

where $\mathrm{Top}(Q)$ represents the topic set of Q, whose definition can be found in [8], and S is the following set denoting "individuals that enter into a mutual R relation":

$$S = \{x : \exists X[x \in X \wedge |X| \geq 2 \wedge X^2 - Id_X^2 \subseteq R^*]\} \tag{17}$$

In (16), the disjunct $\mathrm{Top}(Q) \cap S$ is to handle sentences like "Nobody hit each other" whose denotation does not require a witness set, whereas the conjunct $Q(S) = 1$ corresponds to the "counting operator" discussed in [2] and is used to handle sentences with downward monotonic or non-monotonic type $\langle 1 \rangle$ GQs. It can be shown that when Q is upward monotonic, then (16) reduces to (4) and so (4) is just a special case of (16).

We can now determine the denotation of reciprocal sentences with downward monotonic or non-monotonic type $\langle 1 \rangle$ GQs. For example, by replacing $\text{AND}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})$ with EXACTLY $3(\mathbf{person})$ in (11) (and using (16) as the truth condition of EACH OTHER), we obtain the denotation of the internal reading of the sentence "Exactly three persons read the same books".

In this paper, I will also discuss how to modify the aforesaid framework to cover several types of sentences that are different in one way or another from the ones discussed above. The first type of sentences are those with a weaker reciprocal meaning such as "John, Peter and Mary live in adjacent buildings". Instead of using the truth condition given in (4) which is too strong for this sentence (because three buildings cannot be adjacent to each other in the sense given in (4) unless they are arranged in a circle), I propose to change the inequality $X^2 - Id_X^2 \subseteq R^*$ in (4) to $X^2 - Id_X^2 \subseteq (R^+)^*$ where R^+ represents the transitive closure of R, i.e. the smallest transitive relation containing R.

The second type of sentences are those in which the reciprocal items appear in a syntactic position other than object such as "The same waiter served John, Peter and Mary" in which "the same" appears in the 1st argument position (i.e. subject) of the verb "served". To determine the denotation of this sentence, I propose to change $R(u, w)$ and $R(v, w)$ in (9) to $R(w, u)$ and $R(w, v)$, respectively, so that the variable w representing the noun modified by "the same" now appears in the 1st argument position of R. The modified GNP will now be denoted $(\text{THE SAME AS})_1$.

The third type of sentences are those with more than one reciprocal items such as "John, Peter and Mary read the same books and different magazines". To determine the denotation of this sentence, I propose that we first write out the denotation of "read the same books as and read different magazines than" (with two occurrences of "read") by using AND. By using λ-abstraction to abstract out "read" to obtain a function that maps a binary predicate to another binary predicate, and then applying this function back to **read**, we then obtain the denotation of "read the same books as and different magazines than" (with one occurrence of "read"). The remaining steps of determining the denotation of the aforesaid sentence is as usual.

In the formal framework introduced above, I have been explicitly using the GNP EACH OTHER in the denotation of the sentences discussed above. However, the surface syntactic structure of (1), for instance, does not contain "each other". To further refine the framework to make it more in line with the surface syntactic structure, I propose to introduce a new GNP given below:

$$(\textsc{internal the same})_2 = \lambda X \lambda R \lambda Q[\textsc{each other}((\textsc{the same as})_2(X)(R))(Q)] \tag{18}$$

This new GNP is of type $\langle 1, 2 : \langle 1 \rangle \rangle$. By using this new GNP, we can now write down the denotation of (1) as follows:

$$(\textsc{internal the same})_2(\mathbf{book})(\mathbf{read})(\textsc{and}(I_{\mathbf{j}}, I_{\mathbf{p}}, I_{\mathbf{m}})) \tag{19}$$

Note that this expression does not contain EACH OTHER, which is in line with the surface syntactic structure of (1). Moreover, one can show that by applying the denotation of this new GNP given in (18) to the expression above, we obtain (12), which is the correct truth condition of (1).

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